

Important Properties:

- When a polynomial has four terms, common factors can sometimes be used to factor by grouping.
- Recall the formula for the difference of two squares:

$$\boxed{x^2 - y^2 = (x - y)(x + y)}$$

- It does not matter the order that you list the factors. For example,

$$(3x - 2)(x + 1) = (x + 1)(3x - 2).$$

- You can always check your answer by multiplication.

Common Mistakes to Avoid:

- Recall that the sum of two squares does not factor.
- Sometimes the current order does not lead to a common factor. If this happens try rearranging the terms. Do not assume that this means that the expression cannot be factored.
- Be on the lookout for the difference of squares, the difference of cubes and the sum of cubes. Remember that these can be factored further.

PROBLEMSFactor completely.

1. $2x^3 + 3x^2 - 8x - 12$

$$\begin{aligned} & \underbrace{2x^3 + 3x^2} - \underbrace{8x - 12} \\ & x^2(2x + 3) - 4(2x + 3) \\ & (2x + 3)(x^2 - 4) \\ & \boxed{(2x + 3)(x - 2)(x + 2)} \end{aligned}$$

2. $3x^3 - 2x^2 - 3x + 2$

$$\begin{aligned} & \underbrace{3x^3 - 2x^2} - \underbrace{3x + 2} \\ & x^2(3x - 2) - (3x - 2) \\ & (3x - 2)(x^2 - 1) \\ & \boxed{(3x - 2)(x - 1)(x + 1)} \end{aligned}$$

3. $12x^3 - 16x^2 + 3x - 4$

$$\frac{\underbrace{12x^3 - 16x^2} + \underbrace{3x - 4}}{4x^2(3x - 4) + (3x - 4)}$$

$$\boxed{(3x - 4)(4x^2 + 1)}$$

4. $5x^3 - x^2 + 20x - 4$

$$\frac{\underbrace{5x^3 - x^2} + \underbrace{20x - 4}}{x^2(5x - 1) + 4(5x - 1)}$$

$$\boxed{(5x - 1)(x^2 + 4)}$$

5. $24x^3 - 4x^2 - 6x + 1$

$$\frac{\underbrace{24x^3 - 4x^2} - \underbrace{6x + 1}}{4x^2(6x - 1) - (6x - 1)}$$

$$\frac{(6x - 1)(4x^2 - 1)}{\boxed{(6x - 1)(2x - 1)(2x + 1)}}$$

6. $18x^3 - 27x^2 + 8x - 12$

$$\frac{\underbrace{18x^3 - 27x^2} + \underbrace{8x - 12}}{9x^2(2x - 3) + 4(2x - 3)}$$

$$\boxed{(2x - 3)(9x^2 + 4)}$$

7. $2x^3 + x^2 + 50x + 25$

$$\frac{\underbrace{2x^3 + x^2} + \underbrace{50x + 25}}{x^2(2x + 1) + 25(2x + 1)}$$

$$\boxed{(2x + 1)(x^2 + 25)}$$

8. $10x^2 - 12y + 15x - 8xy$

NOTE: The current order does not lead to a common factor. Therefore, we must first rearrange the terms.

$$\frac{10x^2 - 12y + 15x - 8xy}{10x^2 - 8xy - 12y + 15x}$$

$$\frac{2x(5x - 4y) + 3(-4y + 5x)}{2x(5x - 4y) + 3(5x - 4y)}$$

$$\boxed{(5x - 4y)(2x + 3)}$$

9. $10x^3 - 2x^2y^2 - 5xy + y^3$

NOTE: The current order does not lead to a common factor. Therefore, we must first rearrange the terms.

$$\frac{10x^3 - 2x^2y^2 - 5xy + y^3}{10x^3 - 5xy - 2x^2y^2 + y^3}$$

$$\frac{5x(2x^2 - y) - y^2(2x^2 - y)}{\boxed{(2x^2 - y)(5x - y^2)}}$$