Definitions:

- **Square root of** a: The square root of a, denoted $\sqrt{a}$, is the number whose square is a. In other words,
$$\sqrt{a} = b \quad \text{means} \quad b^2 = a.$$
- n**-th root of** a: The n-th root of a, denoted $\sqrt[n]{a}$, is a number whose n-th power equals a. In other words,
$$\sqrt[n]{a} = b \quad \text{means} \quad b^n = a.$$
The number n is called the **index**.

Important Properties:

- When adding and subtracting radicals, you may only combine radicals with the same index and exactly the same quantity under the radical. For example, $3\sqrt{7} + 5\sqrt{7} = 8\sqrt{7}$. However, $3\sqrt{7}$ and $5\sqrt[3]{7}$ cannot be combined since their indices are different. Likewise, $3\sqrt{7}$ and $5\sqrt{3}$ cannot be combined because the quantity under the radical is different.

Common Mistakes to Avoid:

- The n-th root of a sum does NOT equal the sum of the n-th roots. In other words,
$$\boxed{\sqrt[n]{a+b} \neq \sqrt[n]{a} + \sqrt[n]{b}.}$$
- When combining like radical terms, keep the radical part the same. Do NOT add the radical parts together. For example, $3\sqrt{7} + 5\sqrt{7} \neq 8\sqrt{14}$. Instead,
$$3\sqrt{7} + 5\sqrt{7} = 8\sqrt{7}.$$

PROBLEMS

Perform the indicated operations and simplify.

1. $7\sqrt{11} + 5\sqrt{11} - 4\sqrt{11}$

$$7\sqrt{11} + 5\sqrt{11} - 4\sqrt{11} = \boxed{8\sqrt{11}}$$

2. $3\sqrt{5} - 7\sqrt{2} + 6\sqrt{2}$

$$3\sqrt{5} - 7\sqrt{2} + 6\sqrt{2} = \boxed{3\sqrt{5} - \sqrt{2}}$$

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3. $\sqrt{\dfrac{3}{16}} + \sqrt{\dfrac{75}{4}}$

$$\begin{aligned}
\sqrt{\frac{3}{16}} + \sqrt{\frac{75}{4}} &= \frac{\sqrt{3}}{\sqrt{16}} + \frac{\sqrt{75}}{\sqrt{4}} \\
&= \frac{\sqrt{3}}{4} + \frac{\sqrt{25 \cdot 3}}{2} \\
&= \frac{\sqrt{3}}{4} + \frac{5\sqrt{3}}{2} \\
&= \frac{\sqrt{3}}{4} + \frac{10\sqrt{3}}{4} \\
&= \boxed{\frac{11\sqrt{3}}{4}}
\end{aligned}$$

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4. $3\sqrt{48} + 5\sqrt{27}$

$$\begin{aligned}
3\sqrt{48} + 5\sqrt{27} &= 3\sqrt{16 \cdot 3} + 5\sqrt{9 \cdot 3} \\
&= 3\sqrt{16}\sqrt{3} + 5\sqrt{9}\sqrt{3} \\
&= 3 \cdot 4\sqrt{3} + 5 \cdot 3\sqrt{3} \\
&= 12\sqrt{3} + 15\sqrt{3} \\
&= \boxed{27\sqrt{3}}
\end{aligned}$$

5. $2x\sqrt[3]{54x^4} - 5\sqrt[3]{16x^7}$

$$\begin{aligned}
2x\sqrt[3]{54x^4} - 5\sqrt[3]{16x^7} &= 2x\sqrt[3]{27 \cdot 2x^3x} - 5\sqrt[3]{8 \cdot 2x^6x} \\
&= 2x\sqrt[3]{27x^3}\sqrt[3]{2x} - 5\sqrt[3]{8x^6}\sqrt[3]{2x} \\
&= 2x \cdot 3x\sqrt[3]{2x} - 5 \cdot 2x^2\sqrt[3]{2x} \\
&= 6x^2\sqrt[3]{2x} - 10x^2\sqrt[3]{2x} \\
&= \boxed{-4x^2\sqrt[3]{2x}}
\end{aligned}$$

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6. $3\sqrt{12} - 7\sqrt{72} + 5\sqrt{50}$

$$\begin{aligned}
3\sqrt{12} - 7\sqrt{72} + 5\sqrt{50} &= 3\sqrt{4 \cdot 3} - 7\sqrt{36 \cdot 2} + 5\sqrt{25 \cdot 2} \\
&= 3\sqrt{4}\sqrt{3} - 7\sqrt{36}\sqrt{2} + 5\sqrt{25}\sqrt{2} \\
&= 3 \cdot 2\sqrt{3} - 7 \cdot 6\sqrt{2} + 5 \cdot 5\sqrt{2} \\
&= 6\sqrt{3} - 42\sqrt{2} + 25\sqrt{2} \\
&= \boxed{6\sqrt{3} - 17\sqrt{2}}
\end{aligned}$$

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7. $-2\sqrt{63} + 2\sqrt{28} + 3\sqrt{7}$

$$\begin{aligned}
-2\sqrt{63} + 2\sqrt{28} + 3\sqrt{7} &= -2\sqrt{9 \cdot 7} + 2\sqrt{4 \cdot 7} + 3\sqrt{7} \\
&= -2\sqrt{9}\sqrt{7} + 2\sqrt{4}\sqrt{7} + 3\sqrt{7} \\
&= -2 \cdot 3\sqrt{7} + 2 \cdot 2\sqrt{7} + 3\sqrt{7} \\
&= -6\sqrt{7} + 4\sqrt{7} + 3\sqrt{7} \\
&= \boxed{\sqrt{7}}
\end{aligned}$$

8. $9x\sqrt{243x^2} - 14\sqrt{108x^2} + 2\sqrt{48x^2}$

$$\begin{aligned}
9x\sqrt{243x^2} - 14\sqrt{108x^2} + 2\sqrt{48x^2} &= 9x\sqrt{81\cdot 3x^2} - 14\sqrt{36\cdot 3x^2} + 2\sqrt{16\cdot 3x^2} \\
&= 9x\sqrt{81x^2}\sqrt{3} - 14\sqrt{36x^2}\sqrt{3} + 2\sqrt{16x^2}\sqrt{3} \\
&= 9x\cdot 9x\sqrt{3} - 14\cdot 6x\sqrt{3} + 2\cdot 4x\sqrt{3} \\
&= 81x^2\sqrt{3} - 84x\sqrt{3} + 8x\sqrt{3} \\
&= \boxed{81x^2\sqrt{3} - 76\sqrt{3}}
\end{aligned}$$

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9. $\dfrac{\sqrt{50}}{3} + \dfrac{3\sqrt{8}}{2} + \dfrac{\sqrt{3}}{\sqrt{4}}$

$$\begin{aligned}
\frac{\sqrt{50}}{3} + \frac{3\sqrt{8}}{2} + \frac{\sqrt{3}}{\sqrt{4}} &= \frac{\sqrt{25\cdot 2}}{3} + \frac{3\sqrt{4\cdot 2}}{2} + \frac{\sqrt{3}}{2} \\
&= \frac{\sqrt{25}\sqrt{2}}{3} + \frac{3\sqrt{4}\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \\
&= \frac{5\sqrt{2}}{3} + \frac{3\cdot 2\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \\
&= \frac{5\sqrt{2}}{3} + \frac{6\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \\
&= \frac{10\sqrt{3}}{6} + 3\sqrt{2} + \frac{3\sqrt{3}}{6} \\
&= \boxed{\frac{13\sqrt{3}}{6} + 3\sqrt{2}}
\end{aligned}$$

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10. $5x\sqrt[3]{54x^3y} - 2\sqrt[3]{250x^6y} + 9x^2\sqrt[3]{128y}$

$$\begin{aligned}
5x\sqrt[3]{54x^3y} - 2\sqrt[3]{250x^6y} + 9x^2\sqrt[3]{128y} &= 5x\sqrt[3]{27\cdot 2x^3y} - 2\sqrt[3]{125\cdot 2x^6y} + 9x^2\sqrt{64\cdot 2y} \\
&= 5x\sqrt[3]{27x^3}\sqrt[3]{2y} - 2\sqrt[3]{125x^6}\sqrt[3]{2y} + 9x^2\sqrt[3]{64}\sqrt[3]{2y} \\
&= 5x\cdot 3x\sqrt[3]{2y} - 2\cdot 5x^2\sqrt[3]{2y} + 9x^2\cdot 4\sqrt[3]{2y} \\
&= 15x^2\sqrt[3]{2y} - 10x^2\sqrt[3]{2y} + 36x^2\sqrt[3]{2y} \\
&= \boxed{41x^2\sqrt[3]{2y}}
\end{aligned}$$