Definition:

- **Complex fraction**: is an expression that has a fractions in the numerator, denominator, or both. For example,

$$\frac{\frac{1}{x}}{x+2}, \qquad \frac{\frac{1}{x}-2}{\frac{1}{y}+2}, \qquad \text{and} \qquad \frac{3-\frac{1}{x}}{\frac{1}{x+2}}$$

  are all complex fractions.

Simplifying a complex fraction (Method 1):

1. Simplify the numerator until you have one single fraction.
2. Simplify the denominator until you have one single fraction.
3. Divide by multiplying the numerator by the reciprocal of the denominator.
4. Simplify the result.

Simplifying a complex fraction (Method 2):

1. Find a common denominator of all the fractions in the numerator and all the fractions in the denominator of the complex fraction.
2. Multiply each term in the numerator and denominator of the complex fraction by the common denominator found in Step 1.
3. Simplify the result.

Common Mistakes to Avoid:

- Be careful when working with rational expressions in the denominator. For example,

$$\frac{1}{\frac{1}{x}+\frac{1}{y}} \neq x+y.$$

  To simplify this correctly,

$$\frac{1}{\frac{1}{x}+\frac{1}{y}} = \frac{1}{\frac{y+x}{xy}} = \frac{xy}{y+x}.$$

- Be careful when working with negative exponents. For example,

$$\frac{1}{x^{-1}+y^{-1}} \neq x+y.$$

  To simplify this correctly, recall $x^{-n} = \frac{1}{x^n}$,

$$\frac{1}{x^{-1}+y^{-1}} = \frac{1}{\frac{1}{x}+\frac{1}{y}}.$$

  We will simplify this in problem #7.

PROBLEMS

Simplify each complex fraction.:

1. $\dfrac{\dfrac{x-2}{x}}{\dfrac{x+4}{x^2}}$

METHOD 1:

$$\frac{\dfrac{x-2}{x}}{\dfrac{x+4}{x^2}} = \frac{x-2}{x} \cdot \frac{x^2}{x+4}$$

$$= \boxed{\frac{x(x-2)}{x+4}}$$

METHOD 2:

The common denominator is x^2.

$$\frac{\dfrac{x-2}{x}}{\dfrac{x+4}{x^2}} = \frac{\dfrac{x^2(x-2)}{x}}{\dfrac{x^2(x+4)}{x^2}}$$

$$= \boxed{\frac{x(x-2)}{x+4}}$$

2. $\dfrac{1+\dfrac{1}{x}}{2}$

METHOD 1:

$$\frac{1+\dfrac{1}{x}}{2} = \frac{\dfrac{x}{x}+\dfrac{1}{x}}{2}$$

$$= \frac{\dfrac{x+1}{x}}{2}$$

$$= \frac{x+1}{x} \cdot \frac{1}{2}$$

$$= \boxed{\frac{x+1}{2x}}$$

METHOD 2:

The common denominator is x.

$$\frac{1+\dfrac{1}{x}}{2} = \frac{x+\dfrac{x}{x}}{2x}$$

$$= \boxed{\frac{x+1}{2x}}$$

3. $\dfrac{\dfrac{2x}{x-3}}{\dfrac{5x^2}{x^2-9}}$

METHOD 1:

$$\frac{\dfrac{2x}{x-3}}{\dfrac{5x^2}{x^2-9}} = \frac{\dfrac{2x}{x-3}}{\dfrac{5x^2}{(x-3)(x+3)}}$$

$$= \frac{2x}{x-3} \cdot \frac{(x-3)(x+3)}{5x^2}$$

$$= \boxed{\frac{2(x+3)}{5x}}$$

METHOD 2:

The common denominator is $(x-3)(x+3)$.

$$\frac{\dfrac{2x}{x-3}}{\dfrac{5x^2}{x^2-9}} = \frac{\dfrac{2x}{x-3}}{\dfrac{5x^2}{(x-3)(x+3)}}$$

$$= \frac{\dfrac{2x(x-3)(x+3)}{x-3}}{\dfrac{5x^2(x-3)(x+3)}{(x-3)(x+3)}}$$

$$= \frac{2x(x+3)}{5x^2}$$

$$= \boxed{\frac{2(x+3)}{5x}}$$

4. $\dfrac{\dfrac{1}{y}+\dfrac{1}{y-1}}{\dfrac{1}{y}-\dfrac{2}{y-1}}$

METHOD 1:

$$\frac{\dfrac{1}{y}+\dfrac{1}{y-1}}{\dfrac{1}{y}-\dfrac{2}{y-1}} = \frac{\dfrac{y-1}{y(y-1)}+\dfrac{y}{y(y-1)}}{\dfrac{y-1}{y(y-1)}-\dfrac{2y}{y(y-1)}}$$

$$= \frac{\dfrac{y-1+y}{y(y-1)}}{\dfrac{y-1-2y}{y(y-1)}}$$

$$= \frac{\dfrac{2y-1}{y(y-1)}}{\dfrac{-1-y}{y(y-1)}}$$

$$= \frac{2y-1}{y(y-1)} \cdot \frac{y(y-1)}{-1-y}$$

$$= \boxed{\frac{2y-1}{-1-y}} \quad \text{OR} \quad \boxed{\frac{-(2y-1)}{1+y}}$$

METHOD 2:

The common denominator is $y(y-1)$.

$$\frac{\dfrac{1}{y}+\dfrac{1}{y-1}}{\dfrac{1}{y}-\dfrac{2}{y-1}} = \frac{\dfrac{y(y-1)}{y}+\dfrac{y(y-1)}{y-1}}{\dfrac{y(y-1)}{y}-\dfrac{2y(y-1)}{y-1}}$$

$$= \frac{(y-1)+y}{(y-1)-2y}$$

$$= \boxed{\frac{2y-1}{-1-y}} \quad \text{OR} \quad \boxed{\frac{-(2y-1)}{1+y}}$$

5. $\dfrac{\dfrac{1}{x}-\dfrac{1}{y}}{\dfrac{1}{y^2}-\dfrac{1}{x^2}}$

METHOD 1:

$$\frac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{y^2}-\frac{1}{x^2}} = \frac{\frac{y}{xy}-\frac{x}{xy}}{\frac{x^2}{x^2y^2}-\frac{y^2}{x^2y^2}}$$

$$= \frac{\frac{y-x}{xy}}{\frac{x^2-y^2}{x^2y^2}}$$

$$= \frac{\frac{y-x}{xy}}{\frac{(x-y)(x+y)}{x^2y^2}}$$

$$= \frac{y-x}{xy} \cdot \frac{x^2y^2}{(x-y)(x+y)}$$

$$= \frac{-(x-y)}{xy} \cdot \frac{x^2y^2}{(x-y)(x+y)}$$

$$= \boxed{\frac{-xy}{x+y}}$$

METHOD 2:

The common denominator is x^2y^2.

$$\frac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{y^2}-\frac{1}{x^2}} = \frac{\frac{x^2y^2}{x}-\frac{x^2y^2}{y}}{\frac{x^2y^2}{y^2}-\frac{x^2y^2}{x^2}}$$

$$= \frac{xy^2-x^2y}{x^2-y^2}$$

$$= \frac{xy(y-x)}{(x-y)(x+y)}$$

$$= \frac{-xy(x-y)}{(x-y)(x+y)}$$

$$= \boxed{\frac{-xy}{x+y}}$$

6. $\dfrac{\dfrac{x+2}{x}+\dfrac{1}{x+2}}{\dfrac{5}{x}+\dfrac{x}{x+2}}$

METHOD 1:

$$\frac{\dfrac{x+2}{x}+\dfrac{1}{x+2}}{\dfrac{5}{x}+\dfrac{x}{x+2}} = \frac{\dfrac{(x+2)(x+2)}{x(x+2)}+\dfrac{x}{x(x+2)}}{\dfrac{5(x+2)}{x(x+2)}+\dfrac{x^2}{x(x+2)}}$$

$$= \frac{\dfrac{(x+2)(x+2)+x}{x(x+2)}}{\dfrac{5(x+2)+x^2}{x(x+2)}}$$

$$= \frac{\dfrac{x^2+5x+4}{x(x+2)}}{\dfrac{x^2+5x+10}{x(x+2)}}$$

$$= \frac{x^2+5x+4}{x(x+2)} \cdot \frac{x(x+2)}{x^2+5x+10}$$

$$= \boxed{\frac{x^2+5x+4}{x^2+5x+10}}$$

METHOD 2:

The common denominator is $x(x+2)$.

$$\frac{\dfrac{x+2}{x}+\dfrac{1}{x+2}}{\dfrac{5}{x}+\dfrac{x}{x+2}} = \frac{\dfrac{x(x+2)(x+2)}{x}+\dfrac{x(x+2)}{x+2}}{\dfrac{5x(x+2)}{x}+\dfrac{x^2(x+2)}{x+2}}$$

$$= \frac{(x+2)(x+2)+x}{5(x+2)+x^2}$$

$$= \frac{x^2+4x+4+x}{5x+10+x^2}$$

$$= \boxed{\frac{x^2+5x+4}{x^2+5x+10}}$$

7. $\dfrac{1}{x^{-1}+y^{-1}}$

METHOD 1:

$$\frac{1}{x^{-1}+y^{-1}} = \frac{1}{\frac{1}{x}+\frac{1}{y}}$$

$$= \frac{1}{\frac{y}{xy}+\frac{x}{xy}}$$

$$= \frac{1}{\frac{y+x}{xy}}$$

$$= 1 \cdot \frac{xy}{y+x}$$

$$= \boxed{\frac{xy}{y+x}}$$

METHOD 2:

The common denominator is xy.

$$\frac{1}{x^{-1}+y^{-1}} = \frac{1}{\frac{1}{x}+\frac{1}{y}}$$

$$= \frac{xy}{\frac{xy}{x}+\frac{xy}{y}}$$

$$= \boxed{\frac{xy}{y+x}}$$

8. $\dfrac{x^{-2}+y^{-2}}{x^{-1}+y^{-1}}$

METHOD 1:

$$\frac{x^{-2}+y^{-2}}{x^{-1}+y^{-1}} = \frac{\frac{1}{x^2}+\frac{1}{y^2}}{\frac{1}{x}+\frac{1}{y}}$$

$$= \frac{\frac{y^2}{x^2y^2}+\frac{x^2}{x^2y^2}}{\frac{y}{xy}+\frac{x}{xy}}$$

$$= \frac{\frac{y^2+x^2}{x^2y^2}}{\frac{y+x}{xy}}$$

$$= \frac{y^2+x^2}{x^2y^2} \cdot \frac{xy}{y+x}$$

$$= \boxed{\frac{y^2+x^2}{xy(y+x)}}$$

METHOD 2:

The common denominator is x^2y^2.

$$\frac{x^{-2}+y^{-2}}{x^{-1}+y^{-1}} = \frac{\frac{1}{x^2}+\frac{1}{y^2}}{\frac{1}{x}+\frac{1}{y}}$$

$$= \frac{\frac{x^2y^2}{x^2}+\frac{x^2y^2}{y^2}}{\frac{x^2y^2}{x}+\frac{x^2y^2}{y}}$$

$$= \frac{y^2+x^2}{xy^2+x^2y}$$

$$= \boxed{\frac{y^2+x^2}{xy(y+x)}}$$