Definitions:

- **Polynomial:** A polynomial in x is any expression which can be written as:

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

  where $a_n, a_{n-1}, \ldots, a_1, a_0$ are integers and $a_n \neq 0$.

- **Degree:** The degree of a polynomial is the highest exponent of the polynomial.
- **Monomial:** is a polynomial with one term.
- **Binomial:** is a polynomial with two terms.
- **Trinomial:** is a polynomial with three terms.

Important Properties:

- **Dividing a Polynomial by a Monomial:** When dividing a polynomial by a monomial, divide each term of the polynomial by the monomial:

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

- When dividing a polynomial by a monomial you will need to recall the Quotient Rule for Exponents: For any nonzero number a and any integers m and n,

$$\boxed{\frac{a^m}{a^n} = a^{m-n}.}$$

  When we divide like bases, we subtract the exponents.

- **Dividing Polynomials:** use long division.
- When a certain power of x is missing it is a good idea to put a zero as a place holder for this term.

Common Mistakes to Avoid:

- Do NOT stop the long division process until the remainder is a polynomial with one degree less than the divisor.

PROBLEMS

Perform the indicated operations and simplify.

1. $\dfrac{6x^2+9x-12}{3x}$

$$\frac{6x^2+9x-12}{3x} = \frac{6x^2}{3x} + \frac{9x}{3x} - \frac{12}{3x}$$
$$= \boxed{2x+3-\frac{4}{x}}$$

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2. $\dfrac{20x^4-12x^3+4x^2-6x+8}{2x}$

$$\frac{20x^4-12x^3+4x^2-6x+8}{2x} = \frac{20x^4}{2x} - \frac{12x^3}{2x} + \frac{4x^2}{2x} - \frac{6x}{2x} + \frac{8}{2x}$$
$$= \boxed{10x^3-6x^2+2x-3+\frac{4}{x}}$$

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3. $\dfrac{8x^4-12x^3-2x^2+2x-7}{2x^2}$

$$\frac{8x^4-12x^3-2x^2+2x-7}{2x^2} = \frac{8x^4}{2x^2} - \frac{12x^3}{2x^2} - \frac{2x^2}{2x^2} + \frac{2x}{2x^2} - \frac{7}{2x^2}$$
$$= \boxed{4x^2-6x-1+\frac{1}{x}-\frac{7}{2x^2}}$$

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4. $\dfrac{32x^4y^5-8x^3y^4+7x^2y^3-24xy^2-12xy^7}{-4x^2y^3}$

$$\frac{32x^4y^5-8x^3y^4+7x^2y^3-24xy^2-12xy^7}{-4x^2y^3} = \frac{32x^4y^5}{-4x^2y^3} - \frac{8x^3y^4}{-4x^2y^3} + \frac{7x^2y^3}{-4x^2y^3} - \frac{24xy^2}{-4x^2y^3} - \frac{12xy^7}{-4x^2y^3}$$
$$= \boxed{-8x^2y^2+2xy-\frac{7}{4}+\frac{6}{xy}+\frac{3y^4}{x}}$$

5. $\dfrac{x^2-5x+6}{x-3}$

$$\begin{array}{r} x-2 \\ x-3\,\overline{)\;x^2-5x+6} \\ \underline{x^2-3x} \\ -2x+6 \\ \underline{-2x+6} \\ 0 \end{array}$$

$$\boxed{\text{quotient} = x-2}$$

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6. $\dfrac{x^3-x^2-2x+6}{x-2}$

$$\begin{array}{r} x^2+x \\ x-2\,\overline{)\;x^3-x^2-2x+6} \\ \underline{x^3-2x^2} \\ x^2-2x \\ \underline{x^2-2x} \\ 6 \end{array}$$

$$\boxed{\text{quotient} = x^2+x, \quad \text{remainder} = 6}$$

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7. $\dfrac{x^3+9x^2-5}{x^2-1}$

$$\begin{array}{r} x+9 \\ x^2+0x-1\,\overline{)\;x^3+9x^2+0x-5} \\ \underline{x^3+0x^2-x} \\ 9x^2+x-5 \\ \underline{9x^2+0x-9} \\ x+4 \end{array}$$

$$\boxed{\text{quotient} = x+9, \quad \text{remainder} = x+4}$$

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8. $\dfrac{3x^4-5x^3-20x-5}{x^2+x+3}$

$$\begin{array}{r} 3x^2-8x-1 \\ x^2+x+3\,\overline{)\;3x^4-5x^3+0x^2-20x-5} \\ \underline{3x^4+3x^2+9x^2} \\ -8x^3-9x^2-20x \\ \underline{-8x^3-8x^2-24x} \\ -x^2+4x-5 \\ \underline{-x^2-x-3} \\ 5x-2 \end{array}$$

$$\boxed{\text{quotient} = 3x^2-8x-1, \quad \text{remainder} = 5x-2}$$