Definition:

- A **linear equation in one variable** can be written in the form

$$ax + b = c$$

for real numbers a, b, and c, with $a \neq 0$.

Important Properties:

- **Addition Property of Equality:** If a, b, and c are real numbers, then

$$a = b \quad \text{and} \quad a + c = b + c$$

are equivalent equations. (That is, you can add or subtract the same quantity on both sides of the equation without changing the solution.)

- **Multiplication Property of Equality:** If a, b, and c are real numbers and $c \neq 0$, then

$$a = b \quad \text{and} \quad ac = bc$$

are equivalent equations. (That is, you can multiply or divide the same nonzero quantity on both sides of the equation without changing the solution.)

Common Mistakes to Avoid:

- When clearing the parentheses in an expression like $7 - (2x - 4)$, remember that the minus sign acts like a factor of -1. After using the distributive property, the sign of *every* term in the parentheses will be changed to give $7 - 2x + 4$.
- To clear fractions from an equation, multiply every term on each side by the lowest common denominator. Remember that $\frac{3x}{2}(x - 2)$ is considered one term, whereas, $\frac{3x^2}{2} - 3x$ is considered two terms. To avoid a mistake, clear all parentheses using the distributive property *before* multiplying every term by the common denominator.
- To preserve the solution to an equation, remember to perform the same operation on **both** sides of the equation.

PROBLEMS

Solve for x in each of the following equations:

1. $9x + 3 = 7x - 2$

$$\begin{aligned} 9x + 3 &= 7x - 2 \\ -3 &\qquad -3 \\ 9x &= 7x - 5 \\ -7x &\qquad -7x \\ 2x &= -5 \\ \frac{2x}{2} &= \frac{-5}{2} \\ x &= \frac{-5}{2} \end{aligned}$$

$$\boxed{x = \frac{-5}{2}}$$

2. $0.8x = 6.0x - 31.2$

$$\begin{aligned} 0.8x &= 6.0x - 31.2 \\ -6.0x &\qquad -6.0x \\ -5.2x &= -31.2 \\ \frac{-5.2x}{-5.2} &= \frac{-31.2}{-5.2} \\ x &= 6 \end{aligned}$$

$$\boxed{x = 6}$$

3. $-x + 8 - x = 3x + 9 - 2$

$$\begin{aligned} -x + 8 - x &= 3x + 9 - 2 \\ -2x + 8 &= 3x + 7 \\ -8 &\qquad -8 \\ -2x &= 3x - 1 \\ -3x &\qquad -3x \\ -5x &= -1 \\ \frac{-5x}{-5} &= \frac{-1}{-5} \\ x &= \frac{1}{5} \end{aligned}$$

$$\boxed{x = \frac{1}{5}}$$

4. $3(x - 7) = -2(2x + 3)$

$$\begin{aligned} 3(x - 7) &= -2(2x + 3) \\ 3x - 21 &= -4x - 6 \\ +4x &\qquad +4x \\ 7x - 21 &= -6 \\ +21 &\qquad +21 \\ 7x &= 15 \\ \frac{7x}{7} &= \frac{15}{7} \\ x &= \frac{15}{7} \end{aligned}$$

$$\boxed{x = \frac{15}{7}}$$

5. $7x - 3(5 - x) = 10$

$$\begin{aligned}
7x - 3(5 - x) &= 10 \\
7x - 15 + 3x &= 10 \\
10x - 15 &= 10 \\
+15 & \quad +15 \\
10x &= 25 \\
\frac{10x}{10} &= \frac{25}{10} \\
x &= \frac{25}{10} \\
x &= \frac{5}{2}
\end{aligned}$$

$$\boxed{x = \frac{5}{2}}$$

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6. $0.2(x + 3) - (x - 1.5) = 0.3(x + 2) - 2.9$

$$\begin{aligned}
0.2(x + 3) - (x - 1.5) &= 0.3(x + 2) - 2.9 \\
0.2x + 0.6 - x + 1.5 &= 0.3x + 0.6 - 2.9 \\
-0.8x + 2.1 &= 0.3x - 2.3 \\
-2.1 & \quad -2.1 \\
-0.8x &= 0.3x - 4.4 \\
-0.3x & \quad -0.3x \\
-1.1x &= -4.4 \\
\frac{-1.1x}{-1.1} &= \frac{-4.4}{-1.1} \\
x &= 4
\end{aligned}$$

$$\boxed{x = 4}$$

7. $6(2x + 8) = 4(3x - 6)$

$$\begin{aligned}
6(2x + 8) &= 4(3x - 6) \\
12x + 48 &= 12x - 24 \\
+24 & \quad +24 \\
12x + 72 &= 12x \\
-12x & \quad -12x \\
72 &= 0
\end{aligned}$$

$$\boxed{\text{No Solution}}$$

NOTE: Whenever the variable disappears and a **false** statement (such as $72 = 0$) results, the equation has no solution.

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8. $10(-2x + 1) = -5(3x - 2) - 5x$

$$\begin{aligned}
10(-2x + 1) &= -5(3x - 2) - 5x \\
-20x + 10 &= -15x + 10 - 5x \\
-20x + 10 &= -20x + 10 \\
-10 & \quad -10 \\
-20x &= -20x \\
+20x & \quad +20x \\
0 &= 0
\end{aligned}$$

$$\boxed{\text{All real numbers}}$$

NOTE: Whenever the variable disappears and a **true** statement (such as $0 = 0$) results, the equation is an identity. An identity is true regardless of the number substituted into the variable. As a result, we write "all real numbers" as our answer.

9. $4(7x-2)+3(2-3x)=3(4x-5)-6$

$$
\begin{aligned}
4(7x-2)+3(2-3x) &= 3(4x-5)-6\\
28x-8+6-9x &= 12x-15-6\\
19x-2 &= 12x-21\\
+2 &\qquad +2\\
19x &= 12x-19\\
-12x &\quad -12x\\
7x &= -19\\
\frac{7x}{7} &= \frac{-19}{7}\\
x &= \frac{-19}{7}
\end{aligned}
$$

$$\boxed{x=\frac{-19}{7}}$$

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10. $-7(2-3x)-4(-2x+5)=7-3(5-2x)$

$$
\begin{aligned}
-7(2-3x)-4(-2x+5) &= 7-3(5-2x)\\
-14+21x+8x-20 &= 7-15+6x\\
29x-34 &= 6x-8\\
+34 &\qquad +34\\
29x &= 6x+26\\
-6x &\quad -6x\\
23x &= 26\\
\frac{23x}{23} &= \frac{26}{23}\\
x &= \frac{26}{23}
\end{aligned}
$$

$$\boxed{x=\frac{26}{23}}$$

11. $\frac{x}{3}+3=\frac{x}{5}-\frac{1}{3}$

NOTE: Multiplying each term by the lowest common denominator of 15 will eliminate all fractions.

$$
\begin{aligned}
\frac{x}{3}+3 &= \frac{x}{5}-\frac{1}{3}\\
15\left(\frac{x}{3}\right)+15(3) &= 15\left(\frac{x}{5}\right)-15\left(\frac{1}{3}\right)\\
\frac{15x}{3}+45 &= \frac{15x}{5}-\frac{15}{3}\\
5x+45 &= 3x-5\\
-45 &\qquad -45\\
5x &= 3x-50\\
-3x &\quad -3x\\
2x &= -50\\
\frac{2x}{2} &= \frac{-50}{2}\\
x &= -25
\end{aligned}
$$

$$\boxed{x=-25}$$

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12. $\frac{2x+3}{7}=\frac{x}{4}-\frac{1}{2}$

NOTE: Multiplying each term by the lowest common denominator of 28 will eliminate all fractions.

$$
\begin{aligned}
\frac{2x}{7}+\frac{3}{7} &= \frac{x}{4}-\frac{1}{2}\\
28\left(\frac{2x}{7}\right)+28\left(\frac{3}{7}\right) &= 28\left(\frac{x}{4}\right)-28\left(\frac{1}{2}\right)\\
\frac{56x}{7}+\frac{84}{7} &= \frac{28x}{4}-\frac{28}{2}\\
8x+12 &= 7x-14\\
-12 &\qquad -12\\
8x &= 7x-26\\
-7x &\quad -7x\\
x &= -26
\end{aligned}
$$

$$\boxed{x=-26}$$

13. $-\dfrac{1}{2}(x-12)+\dfrac{1}{4}(x+2)=x+4$

NOTE: Multiplying each term by the lowest common denominator of 4 will eliminate all fractions.

$$
\begin{array}{rcrcrcrcrr}
-\dfrac{x}{2} & + & \dfrac{12}{2} & + & \dfrac{x}{4} & + & \dfrac{2}{4} & = & x & +\ 4 \\
4\left(-\dfrac{x}{2}\right) & + & 4\left(\dfrac{12}{2}\right) & + & 4\left(\dfrac{x}{4}\right) & + & 4\left(\dfrac{2}{4}\right) & = & 4\,(x) & +\ 4\,(4) \\
\dfrac{-4x}{2} & + & \dfrac{48}{2} & + & \dfrac{4x}{4} & + & \dfrac{8}{4} & = & 4x & +\ 16 \\
-2x & + & 24 & + & x & + & 2 & = & 4x & +\ 16 \\
 & & & - & x & + & 26 & = & 4x & +\ 16 \\
 & & & & & - & 26 & & & -\ 26 \\
 & & & - & x & & & = & 4x & -\ 10 \\
 & & & - & 4x & & & & -4x & \\
 & & & - & 5x & & & = & & -\ 10 \\
 & & & & \dfrac{-5x}{5} & & & = & & \dfrac{-10}{-5} \\
 & & & & & & x & = & 2 &
\end{array}
$$

$$\boxed{x=2}$$