

Definitions:

- **Rational Expression:** is the quotient of two polynomials. For example,

$$\frac{x}{y}, \quad \frac{x+1}{3x-2}, \quad \frac{x^2-3x+4}{x^6-3}$$

are all rational expressions.

- **Lowest terms:** A rational expression is in lowest terms when the numerator and denominator contain no common factors.
- **Reciprocal:** The reciprocal of a rational expression  $\frac{a}{b}$  is given by  $\frac{b}{a}$ . To find the reciprocal, we invert (or flip) the rational expression.

Important Properties:

- **Fundamental Property of Rational Numbers:** If  $\frac{a}{b}$  is a rational number and  $c$  is any nonzero real number, then

$$\frac{ac}{bc} = \frac{a}{b}.$$

We use this property to write rational expressions in lowest terms.

- **To write a rational expression in lowest terms:** Factor both the numerator and denominator completely. Apply the Fundamental Property of Rational Numbers to eliminate the common factors.
- **To multiply rational expressions:** Factor all numerators and denominators as much as possible. Apply the Fundamental Property of Rational Numbers to eliminate the common factors. Multiply remaining factors.
- **To divide rational expressions:** Invert the second rational expression and multiply. In other words, multiply the first rational expression by the reciprocal of the second.
- If the numerator and denominator of a rational expression are opposites then the answer is  $-1$ . This is because if we factor out a  $-1$  from either the numerator or denominator, we have

$$\frac{x-y}{y-x} = \frac{-1(y-x)}{y-x} = -1.$$

Common Mistakes to Avoid:

- Remember that only common *factors* can be divided out. For example,

$$\frac{(x-2)(x-1)}{(x+4)(x-1)} = \frac{x-2}{x+4}; \quad \text{however,} \quad \frac{x+2}{x+5} \neq \frac{2}{5} \quad \text{and} \quad \frac{x+2}{x} \neq 2.$$

- To multiply (or divide) rational expressions, you do NOT need a common denominator.
- $x+y$  and  $y+x$  are NOT opposites of one another. Recall that it does not matter the order in which we add terms together. As a result,

$$\frac{x+y}{y+x} = 1.$$

## PROBLEMS

Simplify each expression in lowest terms.

1.  $\frac{x^2 + 5x + 6}{x^2 - 2x + 15}$

$$\frac{x^2 + 5x + 6}{x^2 - 2x + 15}$$

$$\frac{(x + 3)(x + 2)}{(x - 5)(x + 3)}$$

$$\boxed{\frac{(x + 2)}{(x - 5)}}$$

2.  $\frac{y^2 - 3y - 18}{2x^2 + 7y + 3}$

$$\frac{y^2 - 3y - 18}{2y^2 + 7y + 3}$$

$$\frac{(y - 6)(y + 3)}{(2y + 1)(y + 3)}$$

$$\boxed{\frac{y - 6}{2y + 1}}$$

3.  $\frac{x^2 - 1}{1 - x^3}$

$$\frac{x^2 - 1}{1 - x^3}$$

$$\frac{(x - 1)(x + 1)}{(1 - x)(1 + x + x^2)}$$

$$\frac{-1(1 - x)(x + 1)}{(1 - x)(1 + x + x^2)}$$

$$\boxed{\frac{-(x + 1)}{1 + x + x^2}}$$

4.  $\frac{6 - x - 2x^2}{6x^2 + x - 15}$

$$\frac{6 - x - 2x^2}{6x^2 + x - 15}$$

$$\frac{(3 - 2x)(2 + x)}{(2x - 3)(3x + 5)}$$

$$\frac{-1(2x - 3)(2 + x)}{(2x - 3)(3x + 5)}$$

$$\boxed{\frac{-1(2 + x)}{3x + 5}}$$

5.  $\frac{x^2 - 25}{x^2 - x - 20} \cdot \frac{x^2 + x - 12}{x^2 - 5x + 6}$

$$\frac{x^2 - 25}{x^2 - x - 20} \cdot \frac{x^2 + x - 12}{x^2 - 5x + 6}$$

$$\frac{(x - 5)(x + 5)}{(x - 5)(x + 4)} \cdot \frac{(x + 4)(x - 3)}{(x - 3)(x - 2)}$$

$$\boxed{\frac{x + 5}{x - 2}}$$

6.  $\frac{x^2 - 7x - 30}{x^2 - 6x - 40} \cdot \frac{2x^2 + 5x + 2}{2x^2 + 7x + 3}$

$$\frac{x^2 - 7x - 30}{x^2 - 6x - 40} \cdot \frac{2x^2 + 5x + 2}{2x^2 + 7x + 3}$$

$$\frac{(x - 10)(x + 3)}{(x - 10)(x + 4)} \cdot \frac{(2x + 1)(x + 2)}{(2x + 1)(x + 3)}$$

$$\boxed{\frac{x + 2}{x + 4}}$$

$$7. \frac{6x^2 + 5xy - 6y^2}{12x^2 - 11xy + 2y^2} \cdot \frac{8x^2 - 14xy + 3y^2}{4x^2 - 12xy + 9y^2}$$

$$\frac{6x^2 + 5xy - 6y^2}{12x^2 - 11xy + 2y^2} \cdot \frac{8x^2 - 14xy + 3y^2}{4x^2 - 12xy + 9y^2}$$

$$\frac{(2x + 3y)(3x - 2y)}{(4x - y)(3x - 2y)} \cdot \frac{(2x - 3y)(4x - y)}{(2x - 3y)(2x - 3y)}$$

$$\boxed{\frac{2x + 3y}{2x - 3y}}$$


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$$8. \frac{y^2 + 4y - 21}{y^2 + 3y - 28} \div \frac{y^2 + 14y + 48}{y^2 + 4y - 32}$$

$$\frac{y^2 + 4y - 21}{y^2 + 3y - 28} \div \frac{y^2 + 14y + 48}{y^2 + 4y - 32}$$

$$\frac{y^2 + 4y - 21}{y^2 + 3y - 28} \cdot \frac{y^2 + 4y - 32}{y^2 + 14y + 48}$$

$$\frac{(y + 7)(y - 3)}{(y + 7)(y - 4)} \cdot \frac{(y + 8)(y - 4)}{(y + 6)(y + 8)}$$

$$\boxed{\frac{y - 3}{y + 6}}$$

$$9. \frac{6x^4 - 15x^3}{4x^2 - 6x} \div \frac{45x - 18x^2}{2x^2 - x - 3}$$

$$\frac{6x^4 - 15x^3}{4x^2 - 6x} \div \frac{45x - 18x^2}{2x^2 - x - 3}$$

$$\frac{6x^4 - 15x^3}{4x^2 - 6x} \cdot \frac{2x^2 - x - 3}{45x - 18x^2}$$

$$\frac{3x^3(2x - 5)}{2x(2x - 3)} \cdot \frac{(2x - 3)(x + 1)}{9x(5 - 2x)}$$

$$\frac{-3x^3(5 - 2x)}{2x(2x - 3)} \cdot \frac{(2x - 3)(x + 1)}{9x(5 - 2x)}$$

$$\boxed{\frac{-x(x + 1)}{6}}$$


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$$10. \frac{3x^2y^2 - 12x^2}{y^2 + 10y + 16} \div \frac{xy^2 - 12xy + 35x}{y^2 + 3y - 40}$$

$$\frac{3x^2y^2 - 12x^2}{y^2 + 10y + 16} \div \frac{xy^2 - 12xy + 35x}{y^2 + 3y - 40}$$

$$\frac{3x^2y^2 - 12x^2}{y^2 + 10y + 16} \cdot \frac{y^2 + 3y - 40}{xy^2 - 12xy + 35x}$$

$$\frac{3x^2(y^2 - 4)}{(y + 8)(y + 2)} \cdot \frac{(y + 8)(y - 5)}{x(y^2 - 12y + 35)}$$

$$\frac{3x^2(y - 2)(y + 2)}{(y + 8)(y + 2)} \cdot \frac{(y + 8)(y - 5)}{x(y - 7)(y - 5)}$$

$$\boxed{\frac{3x(y - 2)}{y - 7}}$$