Definition:

- **Radical equation**: is an equation with one or more radical expressions.

Important Properties:

- If both sides of an equation are raised to the same power, all solutions of the original equation are also solutions of the new equation.
- When solving an equation involving a even indexed radical, you must always check your "answers" in the *original* equation. We may end up with one or more extraneous solutions. Extraneous solutions may be introduced when we raise both sides of an equation to an even power.

Solving an equation involving a radical:

1. Isolate the radical. Make sure that one radical term is alone on one side of the equation.
2. Raise both *sides* of the equation to the same power that is the same as the index on the radical.
3. Solve the resulting equation. If it still contains a radical, repeat steps 1 and 2.
4. Check your possible solutions in the original equation. Exclude any that result in a false statement.

Common Mistakes to Avoid:

- Remember to raise both *sides* of the equation to the same power. Do NOT raise each *term* to the same power. For example, if $a+b=c$ then raising both sides to the same power we get $(a+b)^n = c^n$. We do NOT get $a^n + b^n = c^n$.
- Remember that $(x+y)^2 \neq x^2+y^2$. Instead, $(x+y)^2 = x^2+2xy+y^2$. Do not forget the middle term. (Note, this formula arises from foil).
- Do NOT forget to check your answers in the original equation.

PROBLEMS

Solve for x in each of the following equations.

1. $\sqrt{x-3} = 4$

Since the square root is already isolated, we start by squaring both sides and solving.

$$\begin{aligned} \sqrt{x-3} &= 4 \\ \left(\sqrt{x-3}\right)^2 &= 4^2 \\ x-3 &= 16 \\ x &= 19 \end{aligned}$$

We need to check our possible solution in the original equation.

Check: $x = 19$

$$\begin{aligned} \sqrt{19-3} &= 4 \\ \sqrt{16} &= 4 \\ 4 &= 4\star \end{aligned}$$

$$\boxed{x = 4}$$

2. $\sqrt{3x-6} - 2 = 1$

Before we square both sides, we need to isolate the radical on one side of the equation.

$$\begin{aligned} \sqrt{3x-6} - 2 &= 1 \\ \sqrt{3x-6} &= 3 \\ \left(\sqrt{3x-6}\right)^2 &= 3^2 \\ 3x-6 &= 9 \\ 3x &= 15 \\ x &= 5 \end{aligned}$$

Check: $x = 5$

$$\begin{aligned} \sqrt{3(5)-6} - 2 &= 1 \\ \sqrt{15-6} - 2 &= 1 \\ \sqrt{9} - 2 &= 1 \\ 3 - 2 &= 1 \\ 1 &= 1\star \end{aligned}$$

$$\boxed{x = 5}$$

3. $x + \sqrt{x-4} = 4$

We must isolate the radical before squaring both sides.

$$
\begin{aligned}
x + \sqrt{x-4} &= 4 \\
\sqrt{x-4} &= 4 - x \\
\left(\sqrt{x-4}\right)^2 &= (4-x)^2 \\
x - 4 &= 16 - 8x + x^2 \\
x &= x^2 - 8x + 20 \\
0 &= x^2 - 9x + 20 \\
0 &= (x-4)(x-5)
\end{aligned}
$$

$$
\begin{aligned}
x - 4 &= 0 \\
x &= 4
\end{aligned}
$$

$$
\begin{aligned}
x - 5 &= 0 \\
x &= 5
\end{aligned}
$$

Next, we must check each possible solution in the original equation.

Check: $x = 4$

$$
\begin{aligned}
4 + \sqrt{4-4} &= 4 \\
4 + 0 &= 4 \\
4 &= 4\star
\end{aligned}
$$

Check: $x = 5$

$$
\begin{aligned}
5 + \sqrt{5-4} &= 4 \\
5 + 1 &= 4 \\
6 &\neq 4
\end{aligned}
$$

$$\boxed{x = 4}$$

4. $\sqrt{x+4} - \sqrt{x-4} = 2$

We must isolate *one* of the radicals before we can square both sides.

$$
\begin{aligned}
\sqrt{x+4} - \sqrt{x-4} &= 2 \\
\sqrt{x+4} &= \sqrt{x-4} + 2 \\
\left(\sqrt{x+4}\right)^2 &= \left(\sqrt{x-4} + 2\right)^2 \\
x + 4 &= x - 4 + 4\sqrt{x-4} + 4 \\
x + 4 &= x + 4\sqrt{x-4} \\
4 &= 4\sqrt{x-4} \\
(4)^2 &= \left(4\sqrt{x-4}\right)^2 \\
16 &= 16(x-4) \\
16 &= 16x - 64 \\
80 &= 16x \\
5 &= x
\end{aligned}
$$

Check: $x = 5$

$$
\begin{aligned}
\sqrt{5+4} - \sqrt{5-4} &= 2 \\
\sqrt{9} - \sqrt{1} &= 2 \\
3 - 1 &= 2 \\
2 &= 2\star
\end{aligned}
$$

$$\boxed{x = 5}$$

5. $\sqrt[3]{x-2}+4=2$

We must isolate the radical before we can raise both sides of the equation to the third power.

$$\sqrt[3]{x-2}+4=2$$
$$\sqrt[3]{x-2}=-2$$
$$\left(\sqrt[3]{x-2}\right)^3=(-2)^3$$
$$x-2=-8$$
$$x=-6$$

$$\boxed{x=-6}$$

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6. $\sqrt{2x+3}-\sqrt{x-2}=2$

We first need to isolate one of the radicals on one side.

$$\sqrt{2x+3}-\sqrt{x-2}=2$$
$$\sqrt{2x+3}=\sqrt{x-2}+2$$
$$\left(\sqrt{2x+3}\right)^2=\left(\sqrt{x-2}+2\right)^2$$
$$2x+3=x-2+4\sqrt{x-2}+4$$
$$2x+3=x+2+4\sqrt{x-2}$$
$$x+1=4\sqrt{x-2}$$
$$(x+1)^2=\left(4\sqrt{x-2}\right)^2$$
$$x^2+2x+1=16(x-2)$$
$$x^2+2x+1=16x-32$$
$$x^2-14x+33=0$$
$$(x-11)(x-3)=0$$

$$x-11=0 \qquad x-3=0$$
$$x=11 \qquad x=3$$

We now need to check our possible solutions into the original equation.

<u>Check: $x=11$</u>

$$\sqrt{2(11)+3}-\sqrt{(11)-2}=2$$
$$\sqrt{22+3}-\sqrt{9}=2$$
$$\sqrt{25}-\sqrt{9}=2$$
$$5-3=2$$
$$2=2\star$$

<u>Check: $x=3$</u>

$$\sqrt{2(3)+3}-\sqrt{3-2}=2$$
$$\sqrt{6+3}-\sqrt{1}=2$$
$$\sqrt{9}-1=2$$
$$3-1=2$$
$$2=2\star$$

$$\boxed{x=11, \quad x=3}$$

7. $\sqrt{2x+5}+\sqrt{x+2}=5$

$$\begin{aligned}
\sqrt{2x+5}+\sqrt{x+2} &= 5\\
\sqrt{2x+5} &= 5-\sqrt{x+2}\\
\left(\sqrt{2x+5}\right)^2 &= \left(5-\sqrt{x+2}\right)^2\\
2x+5 &= 25-10\sqrt{x+2}+x+2\\
2x+5 &= 27+x-10\sqrt{x+2}\\
x-22 &= -10\sqrt{x+2}\\
(x-22)^2 &= \left(-10\sqrt{x+2}\right)^2\\
x^2-44x+484 &= 100(x+2)\\
x^2-44x+484 &= 100x+200\\
x^2-144x+284 &= 0\\
(x-2)(x-142) &= 0
\end{aligned}$$

$$\begin{aligned} x-2 &= 0\\ x &= 2 \end{aligned}$$

$$\begin{aligned} x-142 &= 0\\ x &= 142 \end{aligned}$$

We now need to check our possible solutions into the original equation.

Check: $x=2$

$$\begin{aligned}
\sqrt{2(2)+5}+\sqrt{2+2} &= 5\\
\sqrt{9}+\sqrt{4} &= 5\\
3+2 &= 5\\
5 &= 5\star
\end{aligned}$$

Check: $x=142$

$$\begin{aligned}
\sqrt{2(142)+5}+\sqrt{142+2} &= 5\\
\sqrt{289}+\sqrt{144} &= 5\\
17+12 &= 5\\
29 &\neq 5
\end{aligned}$$

$$\boxed{x=2}$$

8. $\sqrt{x+3}+\sqrt{2-x}=3$

$$
\begin{aligned}
\sqrt{x+3}+\sqrt{2-x}&=3\\
\sqrt{x+3}&=3-\sqrt{2-x}\\
\left(\sqrt{x+3}\right)^2&=\left(3-\sqrt{2-x}\right)^2\\
x+3&=9-6\sqrt{2-x}+2-x\\
x+3&=11-x-6\sqrt{2-x}\\
2x-8&=-6\sqrt{2-x}\\
(2x-8)^2&=\left(-6\sqrt{2-x}\right)^2\\
4x^2-32x+64&=36(2-x)\\
4x^2-32x+64&=72-36x\\
4x^2+4x-8&=0\\
4(x^2+x-2)&=0\\
4(x+2)(x-1)&=0
\end{aligned}
$$

$$
\begin{aligned}
x+2&=0\\
x&=-2
\end{aligned}
$$

$$
\begin{aligned}
x-1&=0\\
x&=1
\end{aligned}
$$

We now need to check our possible solutions into the original equation.

Check: $x=-2$

$$
\begin{aligned}
\sqrt{-2+3}+\sqrt{2-(-2)}&=3\\
\sqrt{1}+\sqrt{4}&=3\\
1+2&=3\\
3&=3\star
\end{aligned}
$$

Check: $x=1$

$$
\begin{aligned}
\sqrt{1+3}+\sqrt{2-1}&=3\\
\sqrt{4}+\sqrt{1}&=3\\
2=1&=3\\
3&=3\star
\end{aligned}
$$

$$\boxed{x=2,\quad x=1}$$