Definitions:

- **Ratio:** is a quotient of two quantities. The ratio of the number x to the number y is written

$$x \text{ to } y, \quad \frac{x}{y}, \quad \text{ or } \quad x : y$$

- **Proportion:** is a statement that says that two ratios are equal.

Important Properties:

- **Cross Multiplication:** can be used to solve a proportion. If $\frac{x}{y} = \frac{z}{w}$ then $xw = yz$.

- Ratios can be stated in a number of ways. For example, part:part, part:whole, or whole:whole.

- Ratios are similar to fractions in that they can be expressed as an ordered pair of numbers representing two amounts. However, ratios are different from fractions in that fractions are restricted to part-whole relationships, whereas ratios are not.

- There are many ways to set up a proportion.

$$\frac{a}{b} = \frac{c}{d} \quad \text{ if and only if } \quad \frac{b}{a} = \frac{d}{c} \quad \text{ if and only if } \quad \frac{a}{c} = \frac{b}{d}.$$

Common Mistakes to Avoid:

- It is important to pay attention to the units of measure when working with proportions. Make sure that all units are the same. For example: If a turtle travels 5 inches every 10 seconds, then how many feet will it travel in 70 seconds?

$$\frac{5 \text{ inches}}{10 \text{ seconds}} \neq \frac{x \text{ feet}}{70 \text{ seconds}}.$$

NOTE: We will solve this example in problem #10.

PROBLEMS

1. If the ratio of males to females in a class is 3 : 5, what is the ratio of males to total students in the class?

For every 3 males there are 5 females. Therefore, there are 3 males for every 8 students.

$$\boxed{3 : 8}$$

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2. An employee making $28,000$ receives a raise of $1,000$. All other employees in the company are given proportional raises. How much of a raise would an employee making $32,000$ receive?

Let $x =$ the raise an employee making $32,000$ would receive.

$$\frac{1,000}{28,000} = \frac{x}{32,000}$$
$$1,000 \cdot 32,000 = 28,000x$$
$$32,000,000 = 28,000x$$
$$1142.86 = x$$

$$\boxed{\text{The employee received a } \$1,142.86 \text{ raise.}}$$

3. If 6 gallons of premium unleaded gasoline costs $8.88, how much would it cost to completely fill a 15 gallon tank?

Let $x =$ the cost to fill 15 gallon tank.

$$\frac{6 \text{ gallons}}{8.88} = \frac{15 \text{ gallons}}{x}$$
$$6x = 133.2$$
$$x = 22.2$$

$$\boxed{\text{It would cost } \$22.20 \text{ to fill a 15 gallon tank}}$$

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4. Mary has 23 yards of fabric available to make costumes for the school play. If each costume requires $3\frac{1}{4}$ yards of fabric, how many costumes can she make?

Let $x =$ the number of costumes Mary can make.

NOTE: $3\frac{1}{4} = 3.25$

$$\frac{1 \text{ costume}}{3.25 \text{ yards}} = \frac{x \text{ costumes}}{23 \text{ yards}}$$
$$23 = 3.25x$$
$$7.08 = x$$

$$\boxed{\text{Mary can make 7 costumes.}}$$

5. If $1\frac{3}{4}$ cups of flour are required to make 30 cookies, how many cups of flour are required to make 96 cookies.

Let x = the number of cups of flour required for 96 cookies.

NOTE: $1\frac{3}{4} = \frac{7}{4}$.

$$\frac{\frac{7}{4} \text{ cups of flour}}{30 \text{ cookies}} = \frac{x \text{ cups of flour}}{96 \text{ cookies}}$$

$$\frac{7}{4} \cdot 96 = 30x$$

$$168 = 30x$$

$$\frac{168}{30} = x$$

$$5\frac{3}{5} = x$$

$$\boxed{5\frac{3}{5} \text{ cups of flour}}$$

6. On a certain wall map, $\frac{1}{3}$ inch represents 18 miles. If two cities are $2\frac{1}{2}$ apart on the map, what is the actual distance between them?

Let x = the actual distance between the two cities.

NOTE: $2\frac{1}{2} = \frac{5}{2}$.

$$\frac{\frac{1}{3} \text{ inch}}{18 \text{ miles}} = \frac{\frac{5}{2} \text{ inch}}{x \text{ miles}}$$

$$\frac{1}{3}x = 18\left(\frac{5}{2}\right)$$

$$\frac{1}{3}x = 45$$

$$x = 135$$

$$\boxed{\text{The two cities are actually 135 miles apart}}$$

7. A quart of yogurt contains 920 calories. Approximately, how many calories would there be in a 5-ounce serving.

Let x = the number of calories in a 5-ounce serving.

NOTE: one quart = 32 ounces

$$\frac{32 \text{ ounces}}{920 \text{ calories}} = \frac{5 \text{ ounces}}{x \text{ calories}}$$

$$32x = 4600$$

$$x = 143.75$$

$$\boxed{\text{Approximately 144 calories in 5-ounce serving}}$$

8. At a certain school there are 7 men for every 5 women. If there are 420 more men than women, what is the total enrollment?

Let x = the number of women.
$x + 420$ = the number of men.
$2x + 420$ = total number of students.

$$\frac{5 \text{ women}}{7 \text{ men}} = \frac{x \text{ women}}{x + 420 \text{ men}}$$

$$5(x + 420) = 7x$$

$$5x + 2100 = 7x$$

$$2100 = 2x$$

$$1050 = x$$

$$\boxed{\text{2520 students are enrolled}}$$

9. John found out that after working for 9 months he had earned 6 days of vacation time. How many days per year does he earn at this rate?

Let $x =$ the number vacations days per year.

$$\frac{6 \text{ days}}{9 \text{ months}} = \frac{x \text{ days}}{12 \text{ months}}$$
$$72 = 9x$$
$$8 = x$$

John earns 8 days of vacation time per year

10. If a turtle travels 5 inches every 10 seconds, then how many feet will it travel in 70 seconds?

Let $x =$ the *inches* the turtle travels in 70 seconds.

$$\frac{5 \text{ inches}}{10 \text{ seconds}} = \frac{x \text{ inches}}{70 \text{ seconds}}$$
$$5(70) = 10x$$
$$350 = 10x$$
$$35 = x$$

NOTE: Now we convert inches into feet by dividing by 12.

$$\frac{35}{12} = 2\frac{11}{12}$$

The turtle travels $2\frac{11}{12}$ feet in 70 seconds.

11. Cheyenne scored 75 goals during her soccer practice. If her success-to-failure rate is 5 : 4, how many times did she attempt to goal?

Let $x =$ number of attempts to goal.

NOTE: 5 : 4 success-to-failure rate translates to a 5 : 9 goals-to-attempts rate.

$$\frac{5 \text{ goals}}{9 \text{ attempts}} = \frac{75 \text{ goals}}{x \text{ attempts}}$$
$$5x = 9(75)$$
$$5x = 675$$
$$x = 135$$

Cheyenne made 135 attempts to goal