Definition:

- **Rational Expression:** is the quotient of two polynomials. For example,

$$\frac{x}{y}, \qquad \frac{x+1}{3x-2}, \qquad \frac{x^2-3x+4}{x^6-3}$$

are all rational expressions.

Solving equations with rational expressions:

1. Find the common denominator of all rational expressions.
2. Rewrite the equation with the common denominator on all terms.
3. Once you have a common denominator, you can equate the numerators.
4. Solve the resulting equation.
5. Check all answers in the original equation and exclude any that make the denominator zero.

Common Mistakes to Avoid:

- When each side of a rational equation is multiplied by a variable expression, the resulting answers may not satisfy the original equation. Check all answers in the original equation and exclude any that make the denominator zero.
- It is not always necessary to multiply all denominators together to find a common denominator. Factor before finding the common denominator.
- Do NOT equate the numerators of two rational expressions unless the denominators are the same. For example, if

$$\frac{2}{x+2} = \frac{3+x}{x} \quad \text{then} \quad 2 \neq 3+x.$$

However, if

$$\frac{2x}{x(x+2)} = \frac{(3+x)(x+2)}{x(x+2)} \quad \text{then} \quad 2x = (3+x)(x+2).$$

PROBLEMS

1. $\dfrac{1}{x+3} = \dfrac{1}{5-x}$

$$\frac{1}{x+3} = \frac{1}{5-x}$$
$$\frac{5-x}{(x+3)(5-x)} = \frac{x+3}{(x+3)(5-x)}$$
$$5-x = x+3$$
$$5-2x = 3$$
$$-2x = -2$$
$$x = 1$$

Since $x = 1$ does not make any of the denominators in our original problem zero, it is our answer.

$$\boxed{x = 1}$$

---

2. $\dfrac{1}{x} + \dfrac{1}{2} = 3$

$$\frac{1}{x} + \frac{1}{2} = 3$$
$$\frac{2}{2x} + \frac{x}{2x} = \frac{3(2x)}{2x}$$
$$\frac{2+x}{2x} = \frac{6x}{2x}$$
$$2+x = 6x$$
$$2 = 5x$$
$$\frac{2}{5} = x$$

Since $x = \frac{2}{5}$ does not make any of the denominators in our original problem zero, it is our answer.

$$\boxed{x = \frac{2}{5}}$$

3. $\frac{1}{x-2} + \frac{3}{5} = \frac{4}{5x-10}$

$$\begin{aligned}
\frac{1}{x-2} + \frac{3}{5} &= \frac{4}{5x-10} \\
\frac{1}{x-2} + \frac{3}{5} &= \frac{4}{5(x-2)} \\
\frac{5}{5(x-2)} + \frac{3(x-2)}{5(x-2)} &= \frac{4}{5(x-2)} \\
\frac{5+3(x-2)}{5(x-2)} &= \frac{4}{5(x-2)} \\
5+3(x-2) &= 4 \\
5+3x-6 &= 4 \\
3x-1 &= 4 \\
3x &= 5 \\
x &= \frac{5}{3}
\end{aligned}$$

Since $x = \frac{5}{3}$ does not make any of the denominators in our original problem zero, it is our answer.

$$\boxed{x = \frac{5}{3}}$$

---

4. $\frac{1}{x-4} + \frac{x}{x+4} = \frac{14}{x^2-16}$

$$\begin{aligned}
\frac{1}{x-4} + \frac{x}{x+4} &= \frac{14}{x^2-16} \\
\frac{1}{x-4} + \frac{x}{x+4} &= \frac{14}{(x-4)(x+4)} \\
\frac{x+4}{(x+4)(x-4)} + \frac{x(x-4)}{(x+4)(x-4)} &= \frac{14}{(x-4)(x+4)} \\
\frac{x+4+x(x-4)}{(x-4)(x+4)} &= \frac{14}{(x-4)(x+4)} \\
x+4+x(x-4) &= 14 \\
x+4+x^2-4x &= 14 \\
x^2-3x+4 &= 14 \\
x^2-3x-10 &= 0 \\
(x-5)(x+2) &= 0
\end{aligned}$$

Setting each factor equal to zero, we get

$$\begin{aligned} x - 5 &= 0 \\ x &= 5 \end{aligned}$$

$$\begin{aligned} x + 2 &= 0 \\ x &= -2 \end{aligned}$$

Since neither $x = 5$ nor $x = -2$ make any of the denominators in our original problem zero, they are both solutions.

$$\boxed{x = 5, \quad x = -2}$$

---

5. $\dfrac{2}{3x+1} + \dfrac{6x}{3x+1} = \dfrac{1}{x}$

$$\begin{aligned} \frac{2}{3x+1} + \frac{6x}{3x+1} &= \frac{1}{x} \\ \frac{2+6x}{3x+1} &= \frac{1}{x} \\ \frac{x(2+6x)}{x(3x+1)} &= \frac{3x+1}{x(3x+1)} \\ x(2+6x) &= 3x+1 \\ 2x + 6x^2 &= 3x+1 \\ 6x^2 - x - 1 &= 0 \\ (3x+1)(2x-1) &= 0 \end{aligned}$$

Setting each factor equal to zero, we get

$$\begin{aligned} 3x + 1 &= 0 \\ 3x &= -1 \\ x &= -\frac{1}{3} \end{aligned}$$

$$\begin{aligned} 2x - 1 &= 0 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

Since $x = -\frac{1}{3}$ makes one of the denominators in the original problem zero, it cannot be a solution.

$$\boxed{x = \frac{1}{2}}$$

6. $\dfrac{4}{x+1} - \dfrac{5}{x^2+4x+3} = \dfrac{3}{x+3}$

$$\begin{aligned}
\frac{4}{x+1} - \frac{5}{x^2+4x+3} &= \frac{3}{x+3} \\
\frac{4}{x+1} - \frac{5}{(x+1)(x+3)} &= \frac{3}{x+3} \\
\frac{4(x+3)}{(x+1)(x+3)} - \frac{5}{(x+1)(x+3)} &= \frac{3(x+1)}{(x+1)(x+3)} \\
\frac{4(x+3)-5}{(x+1)(x+3)} &= \frac{3(x+1)}{(x+1)(x+3)} \\
4(x+3)-5 &= 3(x+1) \\
4x+12-5 &= 3x+3 \\
4x+7 &= 3x+3 \\
x+7 &= 3 \\
x &= -4
\end{aligned}$$

Since $x = -4$ does not make any of the denominators in our original problem zero, it is our solution.

$$\boxed{x = -4}$$

---

7. $\dfrac{3}{x-2} + \dfrac{21}{x^2-4} = \dfrac{14}{x+2}$

$$\begin{aligned}
\frac{3}{x-2} + \frac{21}{x^2-4} &= \frac{14}{x+2} \\
\frac{3}{x-2} + \frac{21}{(x-2)(x+2)} &= \frac{14}{x+2} \\
\frac{3(x+2)}{(x-2)(x+2)} + \frac{21}{(x-2)(x+2)} &= \frac{14(x-2)}{(x-2)(x+2)} \\
\frac{3(x+2)+21}{(x-2)(x+2)} &= \frac{14(x-2)}{(x-2)(x+2)} \\
3(x+2)+21 &= 14(x-2) \\
3x+6+21 &= 14x-28 \\
3x+27 &= 14x-28 \\
-11x+27 &= -28 \\
-11x &= -55 \\
x &= 5
\end{aligned}$$

Since $x = 5$ does not make any of the denominators in our original problem zero, it is our solution.

$$\boxed{x = 5}$$

8. $\dfrac{x}{2x-4} - 2 = \dfrac{1}{x-2}$

$$\frac{x}{2x-4} - 2 = \frac{1}{x-2}$$
$$\frac{x}{2(x-2)} - 2 = \frac{1}{x-2}$$
$$\frac{x}{2(x-2)} - \frac{2\cdot 2(x-2)}{2(x-2)} = \frac{2}{2(x-2)}$$
$$\frac{x-4(x-2)}{2(x-2)} = \frac{2}{2(x-2)}$$
$$x-4(x-2) = 2$$
$$x-4x+8 = 2$$
$$-3x+8 = 2$$
$$-3x = -6$$
$$x = 2$$

Since $x = 2$ makes our denominator zero, it cannot be a solution.

$$\boxed{\text{No solution}}$$

9. $\dfrac{3}{2x+1} - \dfrac{x+1}{2x^2+7x+3} = \dfrac{5}{x+3}$

$$\frac{3}{2x+1} - \frac{x+1}{2x^2+7x+3} = \frac{5}{x+3}$$

$$\frac{3}{2x+1} - \frac{x+1}{(2x+1)(x+3)} = \frac{5}{x+3}$$

$$\frac{3(x+3)}{(2x+1)(x+3)} - \frac{(x+1)}{(2x+1)(x+3)} = \frac{5(2x+1)}{(2x+1)(x+3)}$$

$$\frac{3(x+3)-(x+1)}{(2x+1)(x+3)} = \frac{5(2x+1)}{(2x+1)(x+3)}$$

$$3(x+3)-(x+1) = 5(2x+1)$$

$$3x+9-x-1 = 10x+5$$

$$2x+8 = 10x+5$$

$$-8x+8 = 5$$

$$-8x = -3$$

$$x = \frac{3}{8}$$

Since $x = \frac{3}{8}$ does not make any of the denominators in our original equation zero, it is our solution.

$$\boxed{x = \frac{3}{8}}$$