Important Properties:

- **Zero Product Property:** If a and b are real numbers and $a \cdot b = 0$, then either $a = 0$ or $b = 0$.
- Not all equations can be solved by factoring. If an expression cannot be factored then other methods must be used to solve the equation.

Steps for Solving an Equation by Factoring:

1. Rewrite the equation so that one side is zero.
2. Factor the nonzero side completely.
3. Use the Zero Product Property by setting each factor equal to zero.
4. Solve the resulting equations.

Common Mistakes to Avoid:

- The Zero Product Property can only be used when the product is zero. It is **NOT** true, for example, that if $a \cdot b = 6$ then $a = 6$ or $b = 6$.
- Remember that only factors containing variables lead to solutions.
- Although you may divide each side of an equation by a nonzero number, do **NOT** divide each side of an equation by a variable. This may result in you losing a solution.

PROBLEMS

Solve for x by factoring.

1. $x^2 - 5x - 6 = 0$

$$\begin{aligned} x^2 - 5x - 6 &= 0 \\ (x-3)(x-2) &= 0 \end{aligned}$$

Setting each factor equal to zero, we get

$$\begin{aligned} x - 3 &= 0 \\ x &= 3 \end{aligned} \qquad \begin{aligned} x - 2 &= 0 \\ x &= 2 \end{aligned}$$

$\boxed{\text{Answers: } x = 3, \quad x = 2}$

2. $x^2 - 8x + 15 = 0$

$$\begin{aligned} x^2 - 8x + 15 &= 0 \\ (x-3)(x-5) &= 0 \end{aligned}$$

Setting each factor equal to zero, we get

$$\begin{aligned} x - 3 &= 0 \\ x &= 3 \end{aligned} \qquad \begin{aligned} x - 5 &= 0 \\ x &= 5 \end{aligned}$$

$\boxed{\text{Answers: } x = 3, \quad x = 5}$

3. $6x^2 = 4 + 5x$

NOTE: First, we must rewrite the equation so that one side is zero.

$$\begin{aligned} 6x^2 &= 4 + 5x \\ 6x^2 - 5x - 4 &= 0 \\ (3x-4)(2x+1) &= 0 \end{aligned}$$

Setting each factor equal to zero, we get

$$\begin{aligned} 3x - 4 &= 0 \\ 3x &= 4 \\ x &= \frac{4}{3} \end{aligned}$$

$$\begin{aligned} 2x + 1 &= 0 \\ 2x &= -1 \\ x &= -\frac{1}{2} \end{aligned}$$

$$\boxed{\text{Answers: } x = \frac{4}{3}, \quad x = -\frac{1}{2}}$$

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4. $x(2x+3) = 9$

NOTE: We must first rewrite the equation so that one side is zero. This means that we must multiply out the left hand side.

$$\begin{aligned} x(2x+3) &= 9 \\ 2x^2 + 3x &= 9 \\ 2x^2 + 3x - 9 &= 0 \\ (2x-3)(x+3) &= 0 \end{aligned}$$

Setting each factor equal to zero, we get

$$\begin{aligned} 2x - 3 &= 0 \\ 2x &= 3 \\ x &= \frac{3}{2} \end{aligned}$$

$$\begin{aligned} x + 3 &= 0 \\ x &= -3 \end{aligned}$$

$$\boxed{\text{Answers: } x = \frac{3}{2}, \quad x = -3}$$

5. $16x^3 = 9x$

NOTE: Do NOT divide each side by x. Instead, rewrite the equation so that one side is zero.

$$\begin{aligned} 16x^3 &= 9x \\ 16x^3 - 9x &= 0 \\ x(16x^2 - 9) &= 0 \\ x(4x-3)(4x+3) &= 0 \end{aligned}$$

Setting each factor equal to zero, we get

$$x = 0$$

$$\begin{aligned} 4x - 3 &= 0 \\ 4x &= 3 \\ x &= \frac{3}{4} \end{aligned}$$

$$\begin{aligned} 4x + 3 &= 0 \\ 4x &= -3 \\ x &= -\frac{3}{4} \end{aligned}$$

$$\boxed{\text{Answers: } x = 0, \quad x = \frac{3}{4}, \quad x = -\frac{3}{4}}$$

NOTE: Dividing both side of our original problem by x would have resulted in losing the solution $x = 0$.

6. $12x^3 + 4x^2 - 8x = 0$

NOTE: First we factor out the GCF of $4x$.

$$\begin{aligned} 12x^3 + 4x^2 - 8x &= 0 \\ 4x(3x^2 + x - 2) &= 0 \\ 4x(3x - 2)(x + 1) &= 0 \end{aligned}$$

Setting each factor equal to zero, we get

$$\begin{array}{c|c|c} \begin{aligned} 4x &= 0 \\ x &= 0 \end{aligned} & \begin{aligned} 3x - 2 &= 0 \\ 3x &= 2 \\ x &= \frac{2}{3} \end{aligned} & \begin{aligned} x + 1 &= 0 \\ x &= -1 \end{aligned} \end{array}$$

$$\boxed{\text{Answers } x = 0, \quad x = \frac{2}{3}, \quad x = -1}$$

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7. $12x^2 - 63 = -24x$

NOTE: First we rewrite the equation so that one side is zero.

$$\begin{aligned} 12x^2 - 63 &= -24x \\ 12x^2 + 24x - 63 &= 0 \\ 3(4x^2 + 8x - 21) &= 0 \\ 3(2x + 7)(2x - 3) &= 0 \end{aligned}$$

Setting each factor equal to zero, we get

$$\begin{array}{c|c|c} 3 \neq 0 & \begin{aligned} 2x + 7 &= 0 \\ 2x &= -7 \\ x &= -\frac{7}{2} \end{aligned} & \begin{aligned} 2x - 3 &= 0 \\ 2x &= 3 \\ x &= \frac{3}{2} \end{aligned} \end{array}$$

$$\boxed{\text{Answers: } x = -\frac{7}{2}, \quad x = \frac{3}{2}}$$

REMEMBER: Only factors containing a variable lead to a solution.

8. $24x^3 - 4x^2 - 6x + 1 = 0$

NOTE: A polynomial with four terms we factor by grouping.

$$\begin{aligned} \underbrace{24x^3 - 4x^2} - \underbrace{6x + 1} &= 0 \\ 4x^2(6x - 1) - (6x - 1) &= 0 \\ (6x - 1)(4x^2 - 1) &= 0 \\ (6x - 1)(2x - 1)(2x + 1) &= 0 \end{aligned}$$

Setting each factor equal to zero, we get

$$\begin{array}{c|c|c} \begin{aligned} 6x - 1 &= 0 \\ 6x &= 1 \\ x &= \frac{1}{6} \end{aligned} & \begin{aligned} 2x - 1 &= 0 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned} & \begin{aligned} 2x + 1 &= 0 \\ 2x &= -1 \\ x &= -\frac{1}{2} \end{aligned} \end{array}$$

$$\boxed{\text{Answers: } x = \frac{1}{6}, \quad x = \frac{1}{2}, \quad x = -\frac{1}{2}}$$