

Definition:

- **Prime:** We say that a polynomial is prime if it cannot be factored.

Important Properties:

- Difference of Squares:

$$\boxed{x^2 - y^2 = (x - y)(x + y)}$$

- Difference of Cubes:

$$\boxed{x^3 - y^3 = (x - y)(x^2 + xy + y^2)}$$

- Sum of Cubes:

$$\boxed{x^3 + y^3 = (x + y)(x^2 - xy + y^2)}$$

- Remember that it does not matter in what order you list the factors. For example,

$$(2x - 1)(x + 2) = (x + 2)(2x - 1).$$

Common Mistakes to Avoid:

- The sum of two cubes does factor but the sum of two squares does NOT.
- $x^3 + y^3 \neq (x + y)^3$ AND $x^3 - y^3 \neq (x - y)^2$.

PROBLEMSFactor completely.

1. $9x^2 - 4$

$$\begin{array}{c} 9x^2 - 4 \\ (3x)^2 - 2^2 \\ \boxed{(3x - 2)(3x + 2)} \end{array}$$

2. $16x^2 - 25$

$$\begin{array}{c} 16x^2 - 25 \\ (4x)^2 - 5^2 \\ \boxed{(4x - 5)(4x + 5)} \end{array}$$

3. $16x^2 + 9$

$$16x^2 + 9$$

PRIME

4. $81x^2 - 49$

$$81x^2 - 49$$

$$(9x)^2 - 7^2$$

$(9x - 7)(9x + 7)$

5. $16x^2 - y^2$

$$16x^2 - y^2$$

$$(4x)^2 - y^2$$

$(4x - y)(4x + y)$

6. $x^3 - 8$

$$x^3 - 8$$

$$x^3 - 2^3$$

$(x - 2)(x^2 + 2x + 4)$

7. $x^3 - 27$

$$x^3 - 27$$

$$x^3 - 3^3$$

$(x - 3)(x^2 + 3x + 9)$

8. $x^3 + 8$

$$x^3 + 8$$

$$x^3 + 2^3$$

$(x + 2)(x^2 - 2x + 4)$

9. $x^3 - 1$

$$x^3 - 1$$

$(x - 1)(x^2 + x + 1)$

10. $8x^3 - 1$

$$8x^3 - 1$$

$$(2x)^3 - 1^3$$

$$(2x - 1)((2x)^2 + 2x + 1)$$

$(2x - 1)(4x^2 + 2x + 1)$

11. $27x^3 + 1$

$$27x^3 + 1$$

$$(3x)^3 + 1^3$$

$$(3x + 1)((3x)^2 - 3x + 1)$$

$(3x + 1)(9x^2 - 3x + 1)$

12. $64x^3 - y^3$

$$\begin{aligned} & 64x^3 - y^3 \\ & (4x)^3 - y^3 \\ & (4x - y)((4x)^2 + 4xy + y^2) \\ & \boxed{(4x - y)(16x^2 + 4xy + y^2)} \end{aligned}$$

13. $16x^4 - 81$

$$\begin{aligned} & 16x^4 - 81 \\ & (4x^2 - 9)(4x^2 + 9) \\ & \boxed{(2x - 3)(2x + 3)(4x^2 + 9)} \end{aligned}$$

14. $81x^4 - 1$

$$\begin{aligned} & 81x^4 - 1 \\ & (9x^2)^2 - 1 \\ & (9x^2 - 1)(9x^2 + 1) \\ & \boxed{(3x - 1)(3x + 1)(9x^2 + 1)} \end{aligned}$$