SOLVING SYSTEMS OF LINEAR EQUATIONS BY THE SUBSTITUTION METHOD

Definitions:

- **System of linear equations:** consists of two or more linear equations with the same variables.
- **Consistent:** The system is consistent if there is exactly one solution.
- **Inconsistent:** The system is inconsistent if there is no solution. This happens when the two equations represent parallel lines .
- **Dependent:** The system is dependent if there is an infinite number of ordered pairs as solutions. This occurs when the two equations represent the same line.

Steps for the Substitution Method:

1. Choose one of the equations and solve for one variable in terms of the other variable.
2. Substitute the expression from Step 1 into the other equation.
3. Solve the equation from Step 2. (There will be one equation with one variable).
4. Substitute the solution from Step 3 into either of the original equations. This will give the value of the other variable.

Important Properties:

- The Substitution Method is useful when one equation can be solved very quickly for one of the variables.
- If the equation in Step 3 above is a false statement (such as $7 = 2$), then the system is inconsistent.
- If the equation in Step 3 above is a true statement (such as $0 = 0$), then the system is dependent.

Common Mistakes to Avoid:

- Remember that a system of linear equations is not completely solved until values for both x and y are found. To avoid this mistake, write all answers as an ordered pair.
- Remember that all ordered pairs are stated with the $x-$variable first and the $y-$variable second; namely, (x, y).
- If the first equation is used to solve for the variable, substitute it into the second equation. Otherwise, this will incorrectly lead to the statement $0 = 0$.

PROBLEMS

1. Solve

$$\begin{aligned} 2x + y &= 5 \\ 3x + 2y &= -8 \end{aligned}$$

Notice that the first equation can be solved easily for y, giving us

$$\begin{aligned} 2x + y &= 5 \\ y &= -2x + 5 \end{aligned}$$

This is what we will now substitute into the y variable in our second equation. This gives us:

$$\begin{aligned} 3x + 2(-2x + 5) &= -8 \\ 3x - 4x + 10 &= -8 \\ -x + 10 &= -8 \\ -x &= -18 \\ x &= 18 \end{aligned}$$

Next, we need to find the value of our y variable by substituting $x = 18$ into one of the equations. Since we already know that $y = -2x + 5$, substituting in this equation gives us:

$$\begin{aligned} y &= -2(18) + 5 \\ y &= -36 + 5 \\ y &= -31 \end{aligned}$$

Answer: $(18, -31)$

2. Solve

$$\begin{aligned} 4x + 3y &= 10 \\ 2x + y &= 4 \end{aligned}$$

Notice that we can quickly solve for y using the second equation.

$$\begin{aligned} 2x + y &= 4 \\ y &= -2x + 4 \end{aligned}$$

We will now substitute this into the y variable in our first equation.

$$\begin{aligned} 4x + 3(-2x + 4) &= 10 \\ 4x - 6x + 12 &= 10 \\ -2x + 12 &= 10 \\ -2x &= -2 \\ x &= 1 \end{aligned}$$

We now need to find the value of y by substituting $x = 1$ into one of our equations. Since we already have that $y = -2x + 4$, substituting into this equation gives

$$\begin{aligned} y &= -2(1) + 4 \\ y &= -2 + 4 \\ y &= 2 \end{aligned}$$

Answer: $(1, 2)$

3. Solve

$$\begin{aligned} x - y &= -3 \\ 4x + 3y &= -5 \end{aligned}$$

Notice that the first equation can be solved quickly for either x or y. We will solve for x.

$$\begin{aligned} x - y &= -3 \\ x &= y - 3 \end{aligned}$$

We now substitute this into the x variable in our second equation.

$$\begin{aligned} 4(y-3) + 3y &= -5 \\ 4y - 12 + 3y &= -5 \\ 7y - 12 &= -5 \\ 7y &= 7 \\ y &= 1 \end{aligned}$$

We now substitute $y = 1$ into one of our equations in order to find the value of x. Since we already know that $x = y - 3$, substituting $y = 1$ into this equation yields

$$\begin{aligned} x &= 1 - 3 \\ x &= -2 \end{aligned}$$

Answer: $(-2, 1)$

4. Solve

$$\begin{aligned} 2x - y &= 3 \\ -6x + 3y &= 9 \end{aligned}$$

Notice that the first equation can be solved quickly for y.

$$\begin{aligned} 2x - y &= 3 \\ -y &= -2x + 3 \\ y &= 2x - 3 \end{aligned}$$

We now substitute this into the y variable in our second equation.

$$\begin{aligned} -6x + 3(2x-3) &= 9 \\ -6x + 6x - 9 &= 9 \\ -9 &= 9 \end{aligned}$$

Since this is a false statement, the system is inconsistent. Therefore, there is no solution.

Answer: No Solution

5. Solve

$$\begin{aligned} 2x + 3y &= 5 \\ x - 4y &= 6 \end{aligned}$$

Notice that the second equation can be solved easily for x.

$$\begin{aligned} x - 4y &= 6 \\ x &= 4y + 6 \end{aligned}$$

We will now substitute this into the x variable in our first equation.

$$\begin{aligned} 2(4y+6) + 3y &= 5 \\ 8y + 12 + 3y &= 5 \\ 11y + 12 &= 5 \\ 11y &= -7 \\ y &= -\frac{7}{11} \end{aligned}$$

Finally, we need to solve for the x variable by substituting $y = -\frac{7}{11}$ into one of our equations. Since we already know that $x = 4y+6$ substituting into this equation yields

$$\begin{aligned} x &= 4\left(-\frac{7}{11}\right) + 6 \\ x &= -\frac{28}{11} + 6 \\ x &= -\frac{28}{11} + \frac{66}{11} \\ x &= \frac{38}{11} \end{aligned}$$

$$\boxed{\text{Answer: } \left(\frac{38}{11}, -\frac{7}{11}\right)}$$

6. Solve

$$\begin{aligned} 4x + y &= 10 \\ 3x + 2y &= 5 \end{aligned}$$

Notice that the first equation can be easily solve for y.

$$\begin{aligned} 4x + y &= 10 \\ y &= -4x + 10 \end{aligned}$$

We then substitute this into the y variable in the second equation.

$$\begin{aligned} 3x + 2(-4x + 10) &= 5 \\ 3x - 8x + 20 &= 5 \\ -5x + 20 &= 5 \\ -5x &= -25 \\ x &= -5 \end{aligned}$$

Finally, we need to find the value of y by substituting $x = -5$ into one of our equations. Since we already know that $y = -4x + 10$, substituting into this equation gives us

$$\begin{aligned} y &= -4(-5) + 10 \\ y &= 20 + 10 \\ y &= 30 \end{aligned}$$

$$\boxed{\text{Answer: } (-5, 30)}$$