

Include your name, the homework number, and your complete work, including any steps used to obtain the answer. Submit a hard copy - written out legibly or printed - before class.

**Section 1.5 (20 pts)**

4. Let  $P(x, y)$  be the statement “Student  $x$  has taken class  $y$ ,” where the domain for  $x$  consists of all students in your class and for  $y$  consists of all computer science courses at your school. Express each of these quantifications in English. **(3 pt)**

- a)  $\exists x \exists y P(x, y)$                       b)  $\exists x \forall y P(x, y)$                       c)  $\forall x \exists y P(x, y)$   
 d)  $\exists y \forall x P(x, y)$                       e)  $\forall y \exists x P(x, y)$                       f)  $\forall x \forall y P(x, y)$

10. Let  $F(x, y)$  be the statement “ $x$  can fool  $y$ ,” where the domain consists of all people in the world. Use quantifiers to express each of these statements. **(5 pt)**

- a) Everybody can fool Fred.  
 b) Evelyn can fool everybody.  
 c) Everybody can fool somebody.  
 d) There is no one who can fool everybody.  
 e) Everyone can be fooled by somebody.  
 f) No-one can fool both Fred and Jerry.  
 g) Nancy can fool exactly two people.  
 h) There is exactly one person whom everybody can fool.  
 i) No one can fool himself or herself.  
 j) There is someone who can fool exactly one person besides himself or herself.

26. Let  $Q(x, y)$  be the statement “ $x + y = x - y$ .” If the domain for both variables consists of all integers, what are the truth values? **(3 pt)**

- a)  $\exists x Q(x, 2)$                                       b)  $\exists x \exists y Q(x, y)$                                       c)  $\forall x \exists y Q(x, y)$   
 d)  $\exists y \forall x Q(x, y)$                                       e)  $\forall y \exists x Q(x, y)$                                       f)  $\forall x \forall y Q(x, y)$

30. Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

- a)  $\neg \exists y \exists x P(x, y)$                                       b)  $\neg \forall x \exists y P(x, y)$                                       **(2 pt)**  
 c)  $\neg \exists y (Q(y) \wedge \forall x \neg R(x, y))$                                       d)  $\neg \exists y (\exists x R(x, y) \vee \forall x S(x, y))$

**Section 1.6**

4. What rule of inference is used in each of these arguments? **(2 pt)**

- Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.
- It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.
- Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.
- Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.

20. Determine whether these are valid arguments. **(2 pt)**

- If  $x$  is a positive real number, then  $x^2$  is a positive real number. Therefore, if  $a^2$  is positive, where  $a$  is a real number, then  $a$  is a positive real number.
- If  $x^2 \neq 0$ , where  $x$  is a real number, then  $x \neq 0$ . Let  $a$  be a real number with  $a^2 \neq 0$ ; then  $a \neq 0$ .

24. Identify the error or errors in this argument that supposedly shows that if  $\forall x(P(x) \vee Q(x))$  is true then  $\forall xP(x) \vee \forall xQ(x)$  is true. **(1 pt)**

- $\forall x(P(x) \vee Q(x))$  Premise
- $P(c) \vee Q(c)$  Universal instantiation from (1)
- $P(c)$  Simplification from (2)
- $\forall xP(x)$  Universal generalization from (3)
- $Q(c)$  Simplification from (2)
- $\forall xQ(x)$  Universal generalization from (5)
- $\forall x(P(x) \vee \forall xQ(x))$  Conjunction from (4) and (6)

30. Use resolution to show the hypotheses “Allen is a bad boy or Hillary is a good girl” and “Allen is a good boy or David is happy” imply the conclusion “Hillary is a good girl or David is happy.” **(2 pt)**