Section 2.2 Examples

U = {red, orange, yellow, green, blue, indigo, violet}

A = {red, orange, yellow}

B = {yellow, green, blue}

$$A \cup B, A \cap B, A - B\ , \mathrm{B} - A\ , \bar{A}$$

$A \cup B$ = {*red, orange, yellow, green, blue*}

$A \cap B$ = {*yellow*}

$A - B$ = {*red, orange*}

$B - A$ = {*green, blue*}

$\bar{A}$ = {*green, blue, indigo, violet*}

Show that if A and B are sets with $A \subseteq B$, then $A \cup B = B$.

It is always the case that $B \subseteq A \cup B$, so it remains to show that $A \cup B \subseteq B$. But this is clear because if $x \in A \cup B$, then either $x \in A$, in which case $x \in B$ (because we are given $A \subseteq B$) or $x \in B$; in either case $x \in B$. QED.

Prove that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

by giving an element table proof.

A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	$(A \cap B) \cup (A \cap C)$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

Prove the first identity law $A \cup \emptyset = A$ using set-builder notation

$$
\begin{aligned}
A \cup \emptyset &= \{x \mid x \in A \lor x \in \emptyset\} \\
&= \{x \mid x \in A \lor F\} \\
&= \{x \mid x \in A\} \\
&= A
\end{aligned}
$$