	Equivalent									
		Negation	Conjunction	Disjunction (Inclusive OR)	XOR (Exclusive Or)	Implication	Converse of p →q	Inverse of p →q	Contrapositive of p →q	Biconditional
	рq		p∧q	p ∨ q	p ⊕ q	p →q	q →p	¬ p → ¬ q	¬q → ¬p	p ↔ q
2 ⁿ = # rows	ТТ	F	Т	Т	F	Т	т	Т	Т	Т
	TF	Т	F	Т	Т	F	Т	Т	F	F
	FT		F	Т	Т	Т	F	F	Т	F
	FF		F	F	F	Т	Т	Т	Т	Т

Key Concepts:

- Two propositions are <u>equivalent</u> (p=q) if they always have the same truth-value.
- A list of compound propositions is <u>consistent</u> if it's possible to assign truth-values to the atomic propositions such that each compound proposition in the list is true.
- A tautology is a proposition that's always true.
- A <u>contradiction</u> is a proposition that's always false.
- A <u>contingency</u> is a proposition that's neither a tautology nor a contradiction.
- A compound proposition is <u>satisfiable</u> if there is an assignment of truth values to its variables that make it true.
- A <u>argument</u> in propositional logic is a sequence of propositions. All but the final proposition are called <u>premises</u>. The last statement is the <u>conclusion</u>.
- An argument is <u>valid</u> if the premises imply the conclusion.
- An <u>argument form</u> is an argument that is valid no matter what propositions are substituted into its propositional variables.

Ways of expressing $p \rightarrow q$:

- If p, then q
- If p, q
- q unless ¬p
- q if p
- q whenever p
- q follows from p
- p implies q
- p only if q
- q when p
- p is sufficient for q
- q is necessary for p
- It is necessary to q to p
- A necessary condition for p is q
- A sufficient condition for q is p

Ways of expressing $p \leftrightarrow q$:

- p is necessary and sufficient for q
- If *p* then *q*, and conversely
- *p* iff *q*

Law					
De Morgan's	$\neg \exists x P(x) = \forall x \neg P(x) \neg \forall x P(x) = \exists x \neg P(x)$				
	$\neg(p \land q) \equiv \neg p \lor \neg q$	$\neg(p \lor q) \equiv \neg p \land \neg q$			
Double negation	¬(¬p) = p				
Negation	$p \land \neg p = \mathbf{F}$	$p \lor \neg p \equiv \mathbf{T}$			
Identity	p∧ T ≡p	p∨ F ≡p			
Domination	p∧ F ≡F	p ∨T ≡ T			
Idempotent	p∧p≡p	p∨p≡p			
Communative	p∧d≡d∨b	p∧d≡d∧b			
Associative	(p∧q)∧r ≡p∧(q∧r)	(p∨q)∨r ≡p∨(q∨r)			
Distributive	$p \land (q \lor r) = (p \land q) \lor (p \land r)$	$p\vee(q\wedge r) = (p\vee q)\wedge(p\vee r)$			
Absorption	p∨(b∧d) ≡ b	$b \land (b \lor d) \equiv b$			
Implication	$p \rightarrow d = \neg b \wedge d$				
Contrapositive	$p \rightarrow q \equiv \neg q \rightarrow \neg p$				

Universal Quantifier: $\forall x P(x)$

- "For all x, P(x)"
- "For any arbitrary x, P(x)"
- "For every x, P(x)"
- "For each x, P(x)"

Existential Quantifier: $\exists x P(x)$

- "There exists an x such that P(x)"
- "There is an x such that P(x)"
- "For some x, P(x)"
- "There is at least one x such that P(x)"

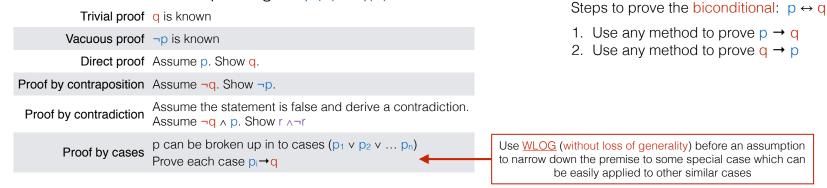
Rules of Inference

Name	Modus Ponens (MP)		Hypothetical Syllogism (HS)		Addition	Simplification	Conjunction	Resolution
Premises	p→q p	p → q ¬q	p → q q → r	p∨q ¬p	р	p∧q	p q	¬p∨r p∨q
Conclusion	q	¬p	p → r	q	p∨q	р	p∧d	q v r

Name	Universal Instantiation (UI)	Universal Generalization (UG)	Existential Instantiation (EI)	Existential Generalization (EG)	Universal Modus Ponens
Premises	∀xP(x)	P(c) for an <u>arbitrary</u> c	∃xP(x)	\cdot P(C) for some element C	$\forall x(P(x) \rightarrow Q(x))$ P(a) where <i>a</i> is a particular element in the domain
Conclusion	P(c)	∀xP(x)	P(c) for some element c	∃xP(x)	Q(a)

Proof methods and techniques

Methods of proving $\forall x p(x) \rightarrow q(x)$



Methods of proving existence: $\exists x P(x)$

Constructive Find an explicit value c for which P(c) is true.

Nonconstructive Assume no c exists which P(c) is true and derive contradiction. Assume $\forall x \neg P(x)$. Show r $\land \neg r$

[there exists one and <u>only one</u> x such that P(x)] Steps to prove unique existence: $\exists x P(x) \land (\forall y P(y) \rightarrow y = x)$

- 1. Prove existence. $\exists x P(x)$
- 2. Prove uniqueness. $\forall y P(y) \rightarrow y=x$