Terminology	Notation	Definition	Meaning
C is a subset of D	C⊆D	∀x(x∈C → x∈D)	Every element of C is also an element of D
C is a proper subset of D	C⊂D	∀x(x∈C → x∈D) ∧ ∃x(x∈D ∧ x∉C)	C⊆D but C≠D. Every element of C is also an element of D, but D has at least one element that C doesn't have
C is equal to D	C=D	∀x(x∈C ↔ x∈D)	C and D have exactly the same elements

N = natural numbers {0,1,2,…}
Z = integers {…,-1,0,1,…}
Z⁺ = positive integers {1,2,3,..}
R = real numbers (ex: 1.5, -π, 40)
R⁺ = positive real numbers (ex: π, 4.2)
Q = rational numbers
U = the universal set
∅ = the empty set

Four different ways to prove A=B

1. Produce a series of identical sets beginning with A and ending with B
2. Prove that both A⊆B and B⊆A.
3. Use set builder notation and propositional logic.
4. Membership tables.

Key Set Concepts:

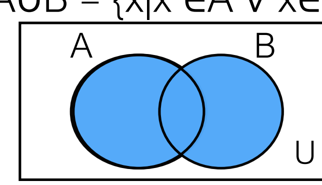
- A set is an unordered collection of objects. It can be described via:
  - roster method: list all elements
  - set builder notation: S = {x | P(x)}
- x ∈ S means x is an element of S.
- x ∉ S means x is not an element of S.
- For every set S, ∅ ⊆ S and S ⊆ S.
- The cardinality of a finite set S, denoted |S|, is the number of distinct elements of S.
- The power set of S, denoted *P*(S), is the set of all subsets of S. If |S|=*n*, then |*P*(S)|=2^n.
- An n-tuple is an ordered collection of *n* objects, denoted as (a_1, a_2, …, a_n)
  - two n-tuples are equal if and only if their corresponding elements are equal
- The Cartesian Product AxB is the set of all ordered pairs between elements of A and elements of B.
  - AxB = {(a,b) | a∈A ∧ b∧B}
- The Cartesian Product A_1xA_2x…xA_n is the set of all ordered n-tuples between elements of A_1, A_2, … A_n.
  - A_1xA_2x…xA_n = {(a_1,a_2,…,a_n) | a_j∈A_j for j=1,2,…n)
- ∀x ∈ S (P(x)) is shorthand for ∀x(x ∈ S → P(x))
- ∀x ∈ S (P(x)) is shorthand for ∀x(x ∈ S → P(x))
- The truth set of some predicate P(x) for a domain D is defined as {x∈D | P(x)}
- The Inclusion-Exclusion principle states that |A∪B|=|A|+|B|-|A∩B|

Set Identities & Operations:

Law	Union	Intersection
Identity	A∪∅=A	A∩U=A
Domination	A∪U=U	A∩∅=∅
Idempotent	A∪A=A	A∩A=A
Double complement	$\overline{(\overline{A})}$=A	
Communative	A∪B=B∪A	A∩B=B∩A
Associative	A∪(B∪C)=(A∪B)∪C	A∩(B∩C)=(A∩B)∩C
Distributive	A∪(B∩C)=(A∩B)∪(A∩C)	A∩(B∪C)=(A∪B)∩(A∪C)
De Morgan's	A∪B = A ∩ B	A∩B = A ∪ B
Absorption	A∪(A∩B)=A	A∩(A∪B)=A
Complement	A∪A = U	A∩A =∅

Union

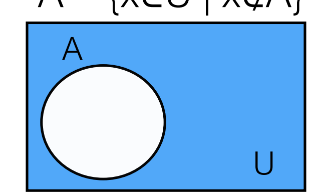
A∪B = {x|x ∈A ∨ x∈B}



$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \ldots \cup A_n$$

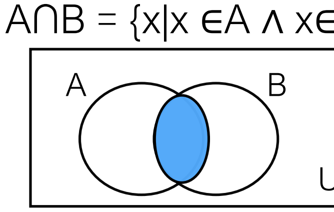
Complement

$\overline{A}$ = {x∈U | x∉A}



Intersection

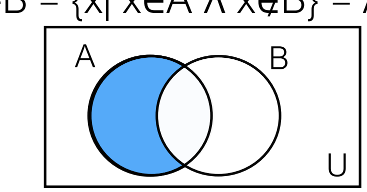
A∩B = {x|x ∈A ∧ x∈B}



$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \ldots \cap A_n$$

Difference

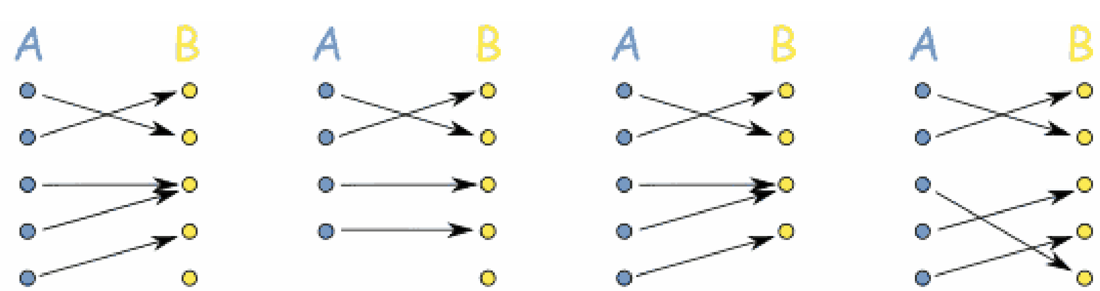
A-B = {x| x∈A ∧ x∉B} = A∩$\overline{B}$



Key Function Concepts:

- A function f from a set A to B, denoted $f: A \to B$, is an an assignment of each element of A to exactly one element of B
  - A is the domain of f
  - B is the codomain of f
  - f(A) is the range of f
  - If $f(a)=b$, then b is the image of a under f, and a is the preimage of b.
- Two functions are equal when they have the same domain, same codomain, and map each element of the domain to the same element of the codomain.
- A function can be represented via:
  - explicit statement of assignments
  - a formula
  - computer program
- Let $f: B \to C$ and $g: A \to B$. The composition of f with g is $f \circ g$: $A \to C$, where $(f \circ g)(a)=f(g(a))$.
- The floor function, denoted $f(x)=\lfloor x \rfloor$, is the largest integer $\leq$ x.
- The ceiling function, denoted $f(x)=\lceil x \rceil$, is the smallest integer $\geq$ x.
- Given a bijective function $f: C \to D$, the inverse $f^{-1}: D \to C$ is defined as $f^{-1}(y)=x$ if and only if $f(x)=y$. No inverse exists unless f is bijective.

- A function $f: C \to D$ is injective (one-to-one) if and only if $\forall x y \in C$ $f(x)=f(y) \to x=y$
- A function $f: C \to D$ is surjective (onto) if and only if $\forall y \in D$ $\exists x \in C$ $f(x)=y$. In this instance, $f(C)=D$.
- A function $f: C \to D$ is bijective (one-to-one correspondence) if and only if f is both injective and surjective.



general function | injective (not surjective) | surjective (not injective) | bijective (injective and surjective)

Key Sequence Concepts:

- A sequence $\{a_n\}$ is a function from the subset of integers to the set S. It provides an ordered list of elements.
- a_n is used to represent f(n), and is called the *n*th term of the sequence.
- A geometric progression is a sequence of the form a, ar, ar^2, ar^3, …, ar^n
- An arithmetic progression is a sequence of the form a, a+d, a+2d,…, a+nd
- A string is a finite sequence of characters from a finite set (an alphabet)
  - the empty string is represented by λ
  - the length of a string is the number of characters in it
- A recurrence relation for a sequence $\{a_n\}$ is an equation that expresses a_n in terms of one or more of previous terms of the sequence
  - It requires initial conditions which specify the terms that precede the first term where the recurrence relation takes effect
  - A sequence is a solution of a recurrence relation if its terms satisfy it
- The Fibonacci sequence f_0, f_1, f_2, … is defined by
  - initial conditions: $f_0=0$ and $f_1=1$
  - recurrence relation: $f_n=f_{n-1}+f_{n-2}$
- Solve the recurrence relation which generates a sequence by finding a closed formula for the nth term of the sequence (doesn't rely on previous terms), using an iterative solution of either forward substitution or backwards substitution
- Sum of terms $a_m, a_{m+1}, \ldots, a_n$ from the sequence $\{a_n\}$ is denoted by $\sum_{j=m}^{n} a_j$

TABLE 2 Some Useful Summation Formulae.

Sum	*Closed Form*
$\sum_{k=0}^{n} ar^k \ (r \neq 0)$	$\frac{ar^{n+1}-a}{r-1}, r \neq 1$
$\sum_{k=1}^{n} k$	$\frac{n(n+1)}{2}$
$\sum_{k=1}^{n} k^2$	$\frac{n(n+1)(2n+1)}{6}$
$\sum_{k=1}^{n} k^3$	$\frac{n^2(n+1)^2}{4}$
$\sum_{k=0}^{\infty} x^k, \lvert x \rvert < 1$	$\frac{1}{1-x}$
$\sum_{k=1}^{\infty} kx^{k-1}, \lvert x \rvert < 1$	$\frac{1}{(1-x)^2}$