MATH 11008:   Plurality with Elimination
Section 1.4

The Plurality with Elimination Method: The candidate with the *MAJORITY* of first place votes is the winner. If no candidate has a majority, then voting is carried out in rounds.

- ROUND 1: Count the first place votes for each candidate. Eliminate the candidate (or candidates if there is a tie) with the fewest first place votes.

- ROUND 2: Cross out the name(s) of the candidate(s) eliminated from the preference schedule. Recount the first place votes. (Remember that when a candidate is eliminated from the preference schedule, in each column the candidates below move up a spot.) If a candidate has a majority of first place votes, declare that candidate the winner. Otherwise, eliminate the candidate with the fewest first place votes.

- ROUND 3, 4, ETC.: Repeat the process, each time eliminating one or more candidates, until there finally is a candidate with a majority of first place votes. This candidate is declared the winner.

○ This method is also known as the Hare Method and instant runoff voting (IRV).

○ For N candidates, requires at most $N - 1$ rounds.

○ Used by the the Academy of Motion Pictures Arts and Sciences Awards to determine the five candidates for each major category. (NOTE: after the five candidates selected, use plurality to determine winner).

○ The International Olympic Committee uses this method when choosing which city gets to host the Olympic Games.

Example 1: The Math Appreciation Society (MAS) is dedicated to the fostering of enjoyment and appreciation of mathematics among college students. During a recent meeting, they have four candidates running for president: Alisha (A), Boris (B), Carmen (C), and Dave (D). Each of the 37 members of the club votes by preference ballot indicating their choices. Below is the preference schedule for this election.

Number of voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

Use the Plurality with Elimination Method to determine the president.

Number of voters					
1st choice					
2nd choice					
3rd choice					
4th choice					

Number of voters					
1st choice					
2nd choice					
3rd choice					
4th choice					

Example 2: A 13-member committee is selecting a chairperson. The 3 candidates are Albert (a), Barbara (b), and Charles (c). Each committee member completely ranked the candidates on a separate ballot. The preference schedule is listed below.

Number of voters	Ranking
4	$a > b > c$
2	$b > c > a$
4	$b > a > c$
3	$c > a > b$

Use the Plurality with Elimination Method to determine the chairperson.

- **The Plurality with Elimination Method and fairness criterion**

  - The Plurality with Elimination Method satisfies the majority criterion; however, it violates the Condorcet criterion.

  - **Monotonicity Criterion:** If candidate X is a winner of an election and, in a reelection, the only changes in the ballots are changes that favor X (and only X), then X should still be the winner.

    * The Plurality with Elimination Method violates the Monotonicity Criterion.

Example 3: Consider an election for club president involving three candidates A, B, and C. For the 29 members of the club, a vote is taken and the the preference schedule for this vote is shown below:

Number of voters	7	8	10	4
1st choice	A	B	C	A
2nd choice	B	C	A	C
3rd choice	C	A	B	B

(a) Determine the winner of the election using the plurality with elimination method.

(b) Unfortunately, club rules were not followed during the vote for club president. Therefore, a recount must take place. Below is the preference schedule for the recount. Use plurality with elimination to determine the club president in the recount.

Number of voters	7	8	14
1st choice	A	B	C
2nd choice	B	C	A
3rd choice	C	A	B

(c) The results of (a) and (b) imply a violation of which fairness criterion?