Topological dynamics of defects: boojums in nematic drops

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The dynamics of creation, annihilation, and transformation of topological defects in a closed system with a vector-type order parameter is studied both theoretically and experimentally. The particular case of a drop of a nematic with varying boundary conditions is considered. In the theoretical part, the concept of a continuously defined topological description of surface defects (e.g., boojums) is introduced. It leads to conservation laws that control the dynamics of the restructuring of the director field in a nematic drop when the boundary conditions vary. It is shown experimentally that when the boundary conditions for a nematic drop are changed, the defects (boojums, hedgehogs, and disclinations) transform into each other and are created from or annihilated into the homogeneous state. All the processes can be described within the scope of the theoretical method developed in the paper.

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I. INTRODUCTION

Ordered systems located in a bounded volume can contain at equilibrium a definite number of structure defects that are stable because of the conditions on the surface. Thus, for example, superfluid ^{3}He-A in a spherical vessel must contain at equilibrium a point vortex—boojum—on the surface.[1] As a result, an undamped superfluid current always circulates around a defect in a ^{3}He-A drop, even at equilibrium. It is also known that in a drop of a nematic liquid crystal (NLC) with normal boundary conditions there must exist at equilibrium another point defect—hedgehog—which can be either inside the drop or on its surface[2,3] (see Fig. 1a).

The boojum and hedgehog are typical examples of two different types of defect in bounded volumes of condensed media. Their topological classification is given in Ref. 4. The defects of the first type exist only on the surface of the system and cannot go into the volume. The defects of the second type exist in the volume, but do not vanish on the surface if they land on it. In the general case, therefore, the defects on the surface comprise a combination of the indicated two types and are characterized by two topological invariants (surface-defect charge and volume-defect charge).

In principle, such defects can be transformed into one another via various processes: for example, a boojum can absorb a hedgehog and be transformed into another boojum. The conservation laws of the topological charges must be satisfied in such processes.

Our task is to investigate theoretically and experimentally the defect topological dynamics connected with their mutual transformations.

From the experimental viewpoint, the most convenient objects are nematic drops with controllable boundary conditions. When the latter change from normal to tangential, the equilibrium state in the drop should change in such a way that the interior defect of Fig. 1a is annihilated and is replaced by two surface defects (Fig. 1b). By varying smoothly the boundary conditions (a procedure that makes this possible is described in Sec. III of the paper) we can observe directly the dynamics of mutual transformations of the defects using a polarization microscope.

In Sec. II we derive theoretically the general topological laws that govern the topological-dynamics processes, using as the example point and linear defects in a nematic drop with arbitrary boundary conditions. To this end, we introduce for surface defects continuously defined topological characteristics that depend on the given boundary conditions. The action of the deduced laws is illustrated by very simple examples.

In Sec. IV we present an experimental confirmation of the action of these laws in nematic drops.

We note that our choice of a concrete model (nematic drop) does not mean that the approach developed cannot be used to solve similar problems for other media and geometries.

II. THEORY OF DEFECTS IN A NEMATIC DROP WITH VARYING BOUNDARY CONDITIONS

§1. Point defects on the surface of a nematic

According to Ref. 4., topologically stable point defects on the boundary of an ordered systems are described by elements of the relative homotopic group $\pi_2(R,\tilde{R})$ where R is

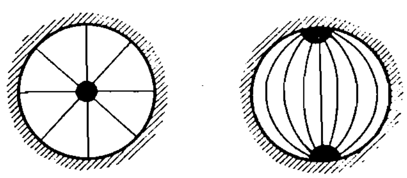

FIG. 1. Distribution of the director field in a nematic drop with normal (a) and tangential (b) boundary conditions: a) radial structure with pointlike volume defect—hedgehog, b) bipolar structure with two surface point defects—boojums.

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the space of degenerate states of the system in the volume, and $\tilde{R}$ is the system of the states that can be possessed by the system on the surface. Usually, and this is the case for NLC, this group can be represented as a product of two groups:

$$\pi_2(R, \tilde{R}) = P \times Q. \tag{1}$$

The elements of group P describe those point defects which can exist only on the surface and cannot go into the interior because of topological restrictions. This type of defect includes boojums in superfluid ^{3}He (Ref. 1); we shall use this designation also for similar defects in other media, including NLC.

Group Q describes point defects that have arrived from the interior and do not vanish on the surface by virtue of topological restrictions imposed by the boundary conditions. In the general case a surface defect is a combination of the indicated defect types and has by the same token two topological charges: an element of group P and an element of group Q.

Let us obtain the corresponding charges for the defects on an NLC surface with arbitrary boundary conditions. Let α_0 be the equilibrium angle between the director $\mathbf{n}$ on the surface of the nematic and the normal ν to it. In this case the region $\tilde{R}$ over which the vector $\mathbf{n}$ varies on the surface is a point at $\alpha_0 = 0$ and a circle at $\alpha_0 \neq 0$; for $\alpha_0 = \pi/2$ diametrally opposite points on this circle are equivalent. Thus,

$$\tilde{R} = \begin{cases} 0, & \alpha_0 = 0, \\ S^1, & 0 < \alpha_0 < \pi/2, \\ S^1/Z_2, & \alpha_0 = \pi/2. \end{cases} \tag{2}$$

Group P, which describes the boojums, is the kernel of the homomorphism $\pi_1(\tilde{R}) \to \pi_1(R)$. Therefore, recognizing that $R = S_2/Z_2$, we find that group P consists of integers m at $\alpha_0 > 0$ and is trivial at $\alpha_0 = 0$. In other words, boojums exist at all conical boundary conditions with $\alpha_0 \neq 0$ and are described by integer topological charges m, which are the numbers of revolutions of the projection of the vector $\mathbf{n}$ on the surface on circling around the boojum along a closed contour located on the boundary [or, in other words it is the index of the planar vector field $\mathbf{n} - \boldsymbol{\nu}(\mathbf{n}\cdot\boldsymbol{\nu})$].

Group Q is the factor-group[4]

$$\pi_2(R)/\mathrm{Im}\,(\pi_2(\tilde{R}) \to \pi_2(R)).$$

In NLC this group coincides with $\pi_2(R)$, i.e., with a group that describes point defects in the interior—hedgehogs. This means that any hedgehog that arrives from the interior does not vanish likewise on the surface, owing to the boundary conditions. Hedgehogs are characterized by integer topological charges N:

$$N = \frac{1}{4\pi} \int_\sigma d\theta d\varphi \mathbf{n} \left[\frac{\partial \mathbf{n}}{\partial \theta} \times \frac{\partial \mathbf{n}}{\partial \varphi} \right] \tag{3}$$

where θ and φ are arbitrary coordinates on a closed surface σ surrounding the point defect in the volume.

Thus, any point defect on the surface is characterized by two charges m and N at $\alpha_0 \neq 0$ and by one charge N at $\alpha_0 = 0$.

To determine the topological charge N of a point defect on the NLC surface it suffices to surround it by a hemisphere $\tilde{\sigma}$ on the volume side and calculate the integral (3) over this hemisphere. The resultant quantity A is connected with m, N, and the projection of $\mathbf{n}$ on the normal $\boldsymbol{\nu}$ near the boojum ($\mathbf{n}\cdot\boldsymbol{\nu} = \cos\alpha_0$) by the relation

$$A_{m,N} = \frac{1}{4\pi} \int_{\tilde{\sigma}} d\theta\, d\varphi \mathbf{n} \left[\frac{\partial \mathbf{n}}{\partial \theta} \times \frac{\partial \mathbf{n}}{\partial \varphi} \right] = \frac{m}{2} (\mathbf{n}\cdot\boldsymbol{\nu} - 1) + N. \tag{4}$$

Here we regard $\mathbf{n}$ not as a director, but as a vector, as is the case in the absence of disclinations in the interior of the nematic, and the vector $\boldsymbol{\nu}$ normal to the surface is regarded is directed outward from the NLC. With the aid of (4) we obtain from A the value of N, since the charge m is determined independently from the index of the vector field $\mathbf{n} - \boldsymbol{\nu}(\mathbf{n}\cdot\boldsymbol{\nu})$ on the surface.

The quantity $A_{m,N}$ is an important characteristic of boojums. If it is an integer ($\alpha_0 = 0$) the defect can break away from the surface or even vanish if $A = 0$. But if $A_{m,N}$ is not an integer, the defect can neither vanish nor go off into the interior.

It is interesting to track the deformation of a surface defect as the boundary conditions change from $\alpha_0 \neq 0$ to $\alpha_0 = 0$. Two cases are possible here: either $A \to 0$ or A tends to an integer different from zero (Figs. 2 and 3). In the latter case the defect goes over gradually into a pure hedgehog, which can then go off into the interior. In the former case the boojum gradually disperses. Ultimately the boojum core, whose radius $\xi_b(\alpha_0)$ depends on α_0, increases with decreasing α_0 and becomes infinite at $\alpha_0 = 0$. The core of a surface defect is the region near the boojum center where, owing to the large gradient energy, the boundary conditions are violated. To find $\xi_b(\alpha_0)$ it is necessary to compare the gradient energy with the surface energy. If the density of the latter is expressed in the form

$$F_s \propto [(\mathbf{n}\cdot\boldsymbol{\nu})^2 - \cos^2 \alpha_0]^2, \tag{5}$$

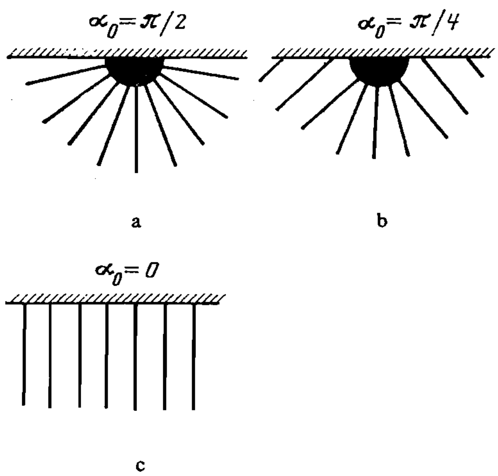


FIG. 2. Dynamics of the vanishing of a boojum with $A = \sin^2(\alpha_0/2)$ and establishment of a defect-free state when the boundary conditions change from tangential to normal. The change of the distribution of the field of $\mathbf{n}$ in the intersection with a vertical plane is shown. The structures are symmetric about the vertical axis ($m = 1$ for Figs. a and b and $m = 0$ for Fig. c).

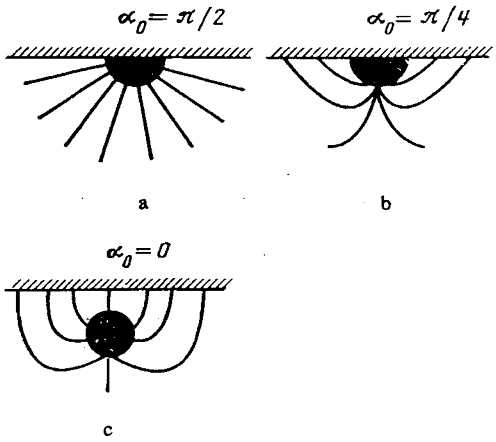


FIG. 3. Dynamics of transformation of a boojum with $A = \cos^2(\alpha_0/2)$ and $m = 1$ into a hedgehog under the same condition as in Fig. 2.

then

$$\xi_b(\alpha_0) \backsim \xi_b/\sin^2\alpha_0, \tag{6}$$

where ξ_b is the dimension of the core at $\alpha_0 = \pi/2$.

§2. Point defects in nematic drops

We consider the nematic drop as a whole. Assume that there are p boojums on its surface and q hedgehogs in its interior. Besides the hedgehogs, the interior may contain annular disclinations, which are topologically equivalent to hedgehogs with integer charges N. These are singular points stretched out into rings, and they do not change the results obtained in the present subsection.

We surround the defects in the interior by a surface σ_2 and the entire surface of the drop, together with the boojums, by a surface σ_1. Obviously, then, the degree of mappings of the surfaces σ_1 and σ_2 on the unit sphere of the vector $\mathbf{n}$ are equal in magnitude and of opposite sign. The degree of mapping of the surface σ_2 is equal to the total charge ΣN_a of the hedgehogs in the interior. The degree of mapping of σ_1 is equal to the sum of the characteristics A for the boojums and the characteristic A_S of the drop surface itself, which differs from zero because of the curvature of the surface. The characteristic A_S is the integral (3) over the drop surface with the locations of the boojums punched out. This integral is equal to the integral, multiplied by $-\mathbf{n}\cdot\mathbf{v}/4\pi$, of the curvature of the surface, which is equal to 4π in the case of a sphere (see Ref. 1). Thus,

$$A_S = -\mathbf{n}\cdot\mathbf{v}. \tag{7}$$

As a result we obtain the equality

$$\sum_{b=1}^{p} A_{m_b,N_b} + A_S + \sum_{a=p+1}^{p+q} N_a = \left(-1+\frac{1}{2}\sum_{b=1}^{p} m_b\right)(\mathbf{n}\cdot\mathbf{v}-1) + \sum_{a=1}^{p+q} N_a - 1 = 0. \tag{8}$$

From (8) follow restrictions on the charges m_b at $\alpha_0 \neq 0$ and on the charges N_a:

$$\sum_b m_b = 2, \quad \sum_{a=1}^{p+q} N_a = 1. \tag{9}$$

The first equality of (9) is the Poincaré theorem: the sum of the indices m_b of a vector field is equal to the Euler characteristic of the drop surface, e.g., to two.

The second equality is a consequence of the Gauss theorem and of the indestructibility of the hedgehogs on the surface. Indeed, since the hedgehogs do not vanish on the boundary, their total charge remains the same as under normal boundary conditions. In this case the total charge N is equal to the integral (3) over the surface of the system with $\mathbf{n} = \mathbf{v}$, and this is none other than the integral, divided by 4π, of the surface curvature, the latter being equal by virtue of the Gauss theorem to half the Euler characteristic, i.e., to unity (see Ref. 1).

Relations (4) and (9) allow us to describe the dynamics of the defects in a nematic drop when the boundary conditions are changed. Under normal boundary conditions ($\alpha_0 = 0$) there are no boojums, and the sum of the charges of the hedgehogs is unity. The equilibrium state of the drop can correspond to one hedgehog (or to a vortex ring having the same charge). Experiment shows that it is more advantageous for the hedgehog to be at the center of the drop.[2,3] Under tangential conditions ($\alpha_0 = \pi/2$) the equilibrium states correspond to two boojums at diametrally opposite points on the drop.[2,3] We consider one of the possible scenarios for a transition from tangential to normal boundary conditions.

Choosing the boojum charges $m_1 = m_2 = 1$, $N_1 = 1$, $N_2 = 0$, we get $A_1 = -A_2 = 1/2$ at $\alpha_0 = \pi/2$. At $\alpha_0 \neq \pi/2$ the values of A_1 and A_2 are no longer equal, inasmuch as according to (4)

$$A_1 = \cos^2(\alpha_0/2), \quad A_2 = -\sin^2(\alpha_0/2).$$

As $\alpha_0 \to 0$, the second boojum disperses, whereas the first is transformed into a hedgehog with charge $N = 1$, which breaks away at $\alpha_0 = 0$ and goes off to the center of the drop.

This simple scenario becomes more complicated if account is taken of the possible formation of surface disclinations.

§3. Disclinations on the surface of a nematic

According to Ref. 4, linear defects on the surface of an ordered medium are described by elements of the relative homotopic group $\pi_1(R,\tilde{R})$. Using the form of R [see (2)] we obtain

$$\pi_1(R,\tilde{R}) = \begin{cases} \pi_1(R) = Z_2, & \alpha_0 \neq \pi/2, \\ 0, & \alpha_0 = \pi/2, \end{cases} \tag{10}$$

i.e., disclinations exist on the surface at all but the tangential boundary conditions, and these disclinations are described by the elements of the same group $\pi_1(R)$ as the disclinations in the interior. Consequently these are those linear disclinations which arrived from the interior and did not vanish on the surface on account of the boundary conditions.

As $\alpha_0 \to \pi/2$, the surface disclinations disperse. This is manifest by the fact that their cores increase to infinity as $\alpha_0 \to \pi/2$. The character of the increase of the cores is obtained, just as in the case of boojums, by comparing the gradient and surface energies (5):

$$\xi_d(\alpha_0)\infty\xi_d/\cos^2\alpha_0, \qquad (11)$$

where ξ_d is the radius of the disclination core on the surface at $\alpha_0 = 0$. Thus, when the boundary conditions are changed from normal to tangential, a surface disclination disperses, and a boojum is enhanced. Conversely, for the reverse transition the boojum is dispersed and the disclination is enhanced, appearing "from nowhere."

We call attention to the fact that topologically speaking nothing prevents a surface disclination from going off to the interior, since it is described by the same element of the homotopic group $\pi_1(R)$ as a volume disclination. Nonetheless, such a move can be difficult. The reason is that entry into the volume calls for formation of the hard core typical of volume disclinations. This calls for great expenditure, particularly large when α_0 is far from zero and the core of the surface disclination is friable.

Let us examine the possible influence of annular disclinations on the dynamics of defects in a drop. We shall not consider the exotic case of an open surface disclination, i.e., an annular disclination part of which is on the surface and part in the volume. Inasmuch as both boojums and hedgehogs can be continuously distributed on a closed surface disclination, this surface has likewise charges m and N. Owing to the curvature of the drop surface, however, the characteristic A of a disclination is expressed in terms of m and N in a complicated manner that depends on the form of the disclination.

We confine ourselves to the simplest case which, as will be seen from Sec. IV, is in fact realized in experiment. Namely, we take one disclination with zero charges $m = 0$ and $N = 0$. In this case, if the disclination is on a geodesic, say on the equator, its characteristic is $A_d = 0$. The disclination divides the drop surface S into two parts, S_+ and S_-, with opposite signs of the values of $\mathbf{n}\cdot\mathbf{v}$:$\mathbf{n}\cdot\mathbf{v} > 0$ on S_+ and $\mathbf{n}\cdot\mathbf{v} < 0$ on S_-. If the disclination is on the equator, the characteristics of these surfaces cancel out, $A_{S_+} + A_{S_-} = 0$, since the areas of S_+ and S_- are equal. Let now the disclination shift away from the equator. By virtue of the conservation of the quantity $A_d + A_{S_+} + A_{S_-} = 0$ we have for the characteristic A_d the expression

$$A_d=(\mathbf{n}\cdot\mathbf{v})_+\frac{S_+}{S}+(\mathbf{n}\cdot\mathbf{v})_-\frac{S_-}{S}. \qquad (12)$$

If the disclination is on a parallel with latitude β, we have according to (12)

$$A_d=\pm\sin\beta\cos\alpha_0.$$

The conservation laws change in the presence of a disclination. Since no account need be taken of $A_d + A_S = 0$ in (8), we have instead of (9)

$$\sum m_+=\sum m_-,$$
$$\sum N=\frac{1}{2}\left(\sum m_+ + \sum m_-\right), \qquad (13)$$

where m_+ and m_- are the charges of boojums located on S_+ and S_-, respectively.

We describe now the second possible scenario of a transition from tangential to normal conditions in the drop, with participation of the disclination. We choose at $\alpha_0 = \pi/2$, as before, a state with two boojums on opposite poles of the drop, with charges $m_1 = m_2 = 1$, $N_1 = 1$, $N_2 = 0$, so that $A_1 = -A_2 = 1/2$. In this scenario, as α_0 decreases smoothly from $\pi/2$, a disclination is produced "out of nothing" on the equator and becomes gradually enhanced. Putting $m_1 = m_-$, $m_2 = m_+$, i.e., $(\mathbf{n}\cdot\mathbf{v})_2 = \cos\alpha_0(\mathbf{n}\cdot\mathbf{v})_1 = -\cos\alpha_0$, we obtain the following characteristics A for the boojums:

$$A_1=\sin^2(\alpha_0/2), \quad A_2=-\sin^2(\alpha_0/2),$$

i.e., in contrast to the preceding scenario, the boojums vary in like fashion.

As α_0 is decreased, the process can follow different paths:

a) The boojums disperse gradually whereas the disclination moves and is squeezed towards one of the poles. According to (12), A_d tends in this case to $\pm\cos\alpha_0 \to \pm 1$, forming a pointlike hedgehog that goes off subsequently into the interior.

b) The disclination contracts to one of the boojums, say the first, forming a defect with $A_2 = \cos^2(\alpha_0/2)$, after which everything follows the first scenario.

Which of the scenarios is realized depends on the actual energy parameters of the system. This can be determined either by an exact calculation of the various textures in the drop, or by experiment.

We proceed now to the results of an experimental investigation of the topological dynamics of the defects in drops of a nematic when the boundary conditions change from strictly tangential to strictly normal and vice versa.

III. EXPERIMENTAL PROCEDURE

We investigated a number of nonylhydrobenzoic-acid esters, a common feature of which is that the molecules have hydrophobic end chains and hydrophilic rigid nuclei. Figure 4 shows the phase diagram and the structural formula of one (n-butoxyphenyl) ester. All the experimental results cited in this paper pertain to just this substance.

Spherical volumes of the nematic were produced by dispersing the latter in the form of droplets of radius 5–50 μm in an isotropic liquid. If this matrix is taken to be glycerin, tangential boundary conditions for $\mathbf{n}$ are ensured on the droplet surfaces. The reason is that the hydroxyl OH groups contained in the glycerin molecules interact more strongly with the hydrophilic nuclei of the nematic molecules than

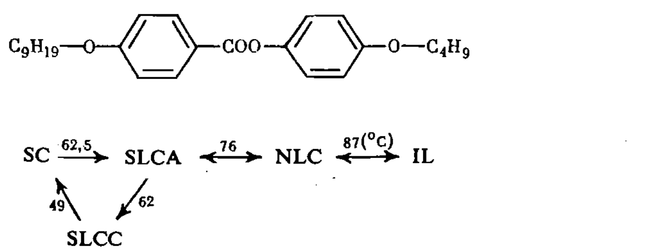


FIG. 4. Structural formula and diagram of states of n-butoxyphenyl ester of nonyloxybenzoic acid.

with their hydrophobic ends. Addition of lecithin solutions to the matrix can change the boundary conditions into normal,[3] for now it is the hydrophobic terminations of the molecules that make the largest contribution to the interaction between the liquid crystal and the matrix (the lecithin molecule is a glycerine molecule in which the OH groups are replaced by hydrophobic chains.[5]

Thus, a glycerin-lecithin matrix of constant composition is potentially capable of setting different boundary conditions, in view of the presence of forces responsible for normal orientation of the NLC and forces that produce tangential orientation. Obviously, the temperature dependences of these forces need not necessarily coincide. It is this idea on which the procedure employed is based: the boundary conditions on the droplet surfaces were set by varying the sample temperature. The procedure ensured uniform and controllable orientation of the NLC at the droplet surface, as well as good reproducibility of the results.

It was observed that for the investigated substances such a matrix sets up strictly tangential boundary conditions in the region preceeding the transition from an NLC to an isotropic liquid, and strictly normal ones in the lower part of the temperature interval of existence of the nematic phase. A smooth lowering of the temperature within the indicated limits led to a smooth variation of the angle α_0 between the director $\mathbf{n}$ and the normal $\boldsymbol{\nu}$ to the droplet surface from $\pi/2$ to 0; the reverse occurs when the sample is heated. That the dependence of α_0 on the temperature T is monotonic is confirmed by measurements of the light transmissivity of a thin (20–40 μm) NLC layer placed between transparent plates coated beforehand by a layer of the indicated matrix (Fig. 5).

From the same measurements one can determine also the $\alpha_0(T)$ dependence quantitatively, but it can be determined more directly from the deviation of the extinction branches in the droplet textures on direction of polarization of nicols as viewed through a microscope (Fig. 5). The method is based on the fact that the extinction branches are localized at those texture spots where the optical axes of the molecules are oriented along the nicol directions.

On the whole, from the known features of the behavior of optically homogeneous media in different investigation regimes[6] permit an unambiguous reconstruction of the distribution of the field of $\mathbf{n}$ in the drops, meaning a determination of the topological characteristics of the defects. By way of example we consider the method of determining the values of m and A for boojums of the type shown in Figs. 2b and 3b.

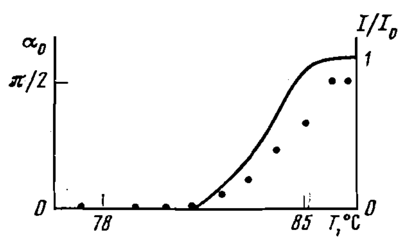


FIG. 5. Dependence of the relative optical transmission I/I_0 (solid curve) and of the angle α_0 on the temperature for NLC drops in a glycerin-lecithin matrix.

The value of m is determined from the number m of the extinction branches that start out from the center of the boojum, if they are observed along the symmetry axis of the structure (from above): in the general case[7]

$$|m| = M/4,$$

and $m > 0$ if the branches remain stationary when the sample is rotated in the horizontal plane, or if they turn in the same direction as the sample.

To determine A it is necessary to determine first, from the inclination of the extinction branches, the value of α_0. It remains next to ascertain the boojum type [with $A = \sin^2(\alpha_0/2)$ or with $A = \cos^2(\alpha_0/2)$]. To this end it is necessary to orient the sample in such a way that the boojum symmetry axis is parallel to the polarization of one of the nicols. Then, as can be easily seen from Figs. 2b and 3b, two extinction branches emerging from the center are observed for the second boojum, while for the boojum with $A = \sin^2(\alpha_0/2)$ there will be none if twist strains are present in the volume (see Sec. IV, §1), or else three and one, respectively, if there are no such strains. This determines A (apart from the sign).

The topological characteristics of a hedgehog and of a disclination are determined analogously. The value of N for boojums is determined from (4).

The texture evolution with changing temperature was observed using an NU-2E polarization microscope with orthoscopic ray arrangement. The temperature was recorded accurate to 0.02 °C and varied at a rate 0.1 °C/min.

IV. EXPERIMENTAL RESULTS AND THEIR DISCUSSION

This section consists of several subsections. §§2 and 3 deal respectively with the defect topological dynamics proper for the transition from tangential to normal boundary conditions and the reverse. They are preceded by §1, in which are discussed bipolar structure realized in drops at strictly tangential boundary conditions. Radial structures (normal conditions of director orientation) are discussed at the end of §2.

§1. Bipolar structure

A bipolar structure is produced in an NLC drop when it is cooled from the isotropic phase, and is characterized by the presence of two surface point defects (boojums) at diametrally opposite poles of the drop (Fig. 6); the boundary conditions are strictly tangential in this case. Similar structures were observed earlier for different types of nematics,[2,3] and it was indicated in Ref. 2 that the director distribution for them is of the form shown in Fig. 1b. Such a structure contains only two types of deformation: transverse and longitudinal bends. The results of the present investigation indicate that bipolar structures can be more complicated and contain twist deformations.

Indeed, when the drop axis is oriented in the horizontal plane along the polarization direction of one of the nicols crossed at right angle, the central part of its structure is not extinguished [as would be the case for the distribution in Fig. 1b (Ref. 2)], and the extinction occurs only when the nicols

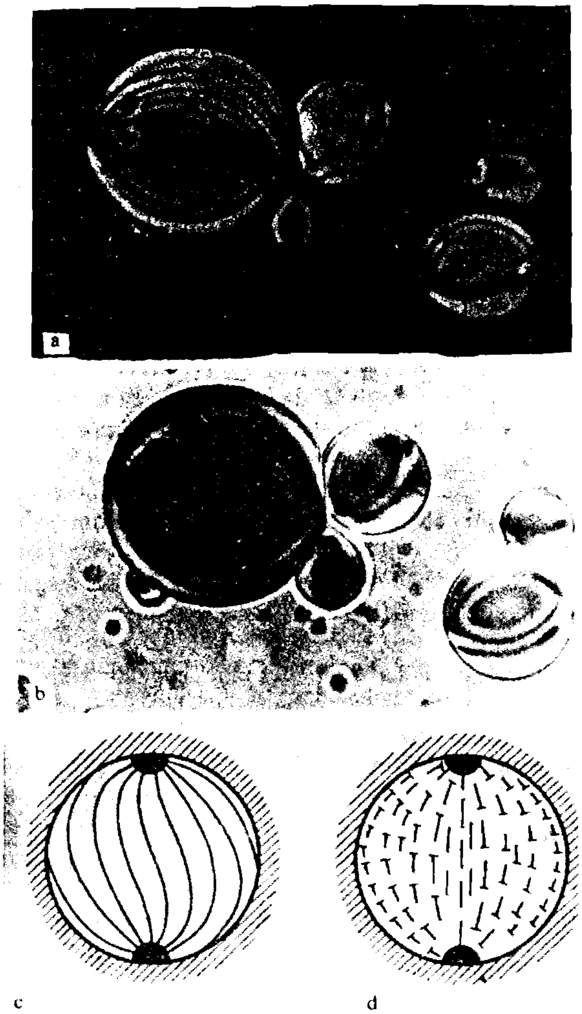


FIG. 6. Bipolar structures in NLC drops with twist deformations: a) nicols crossed at right angle (the polarization directions are oriented along the edges of the photograph); b) nicols crossed at an angle 70°; c) distribution of director field on the drop surface—along loxodromes that cross the meridians at constant angle 10°; d) twist deformation in the distribution of the molecules inside the drop (intersection with vertical plane).

are turned through some relative angle γ different from 90° (Fig. 6, a,b). Connected with γ is the angle γ' of rotation of the plane of polarization of the beam passing through the drop, as well as the angle γ'' between the directions of the director in the lower and upper hemispheres of the surface of the drop (Fig. 6c):

$$\gamma''=\gamma'=\pi/2-\gamma.$$

From the last relation we can determine the angle γ''. For the drops of Fig. 6 we have $\gamma = 70°$ and $\gamma'' = 20°$. The nonzero value of γ'', which is set by the ratio of the Frank elastic constants, attests to the presence of twist deformations inside the nematic, with the director oriented in the central part of the drop along the symmetry axis that joins the boojums. Indeed, in the absence of twists linear defects that go off in the interior would appear near the boojums, in contradiction to the experiment.

The field of **n** on the drop surface is thus distributed along loxodrome helices (Fig. 6c). The corresponding distribution of the molecules inside the drop is shown in Fig. 6d.

The indicated distortions of the bipolar structure, however, do not change its topological characteristics—one of the boojums act as the source of the field of **n**, and the other as the sink; in this case

$$m_1=1,\ A_1={}^1/_2,\ N_1=1;\ m_2=1,\ A_2=-{}^1/_2;\ N_2=0.$$

We examine now how the boojums are annihilated and a hedgehog created when the boundary conditions are changed.

§2. Transition from bipolar to radial structure

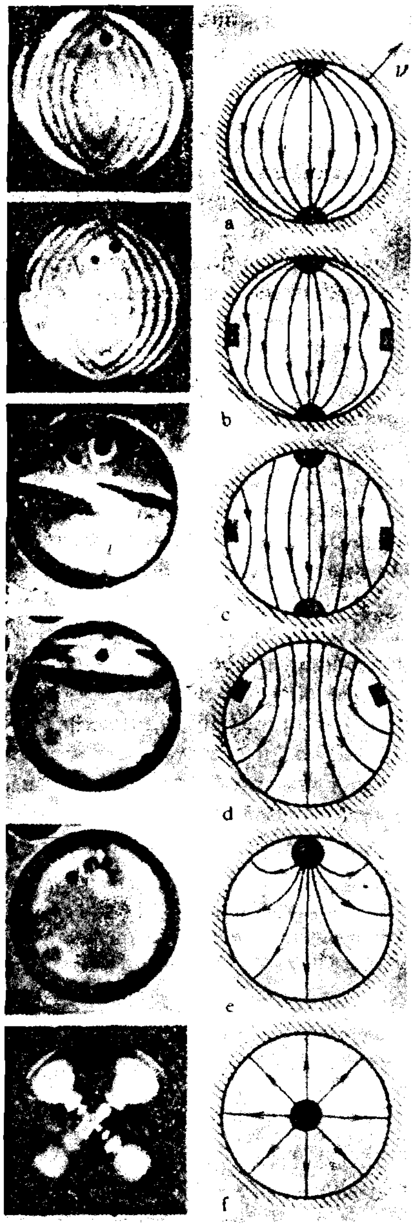
The transition takes place as the equilibrium value of the angle α_0 changes from $\pi/2$ to 0. The experimental results are shown in Fig. 7.

In the initial state we have a purely bipolar structure (Fig. 7a). When α_0 begins to decrease, the orientation of the molecules near the boojums changes in symmetric fashion and, as seen from Fig. 7b, the boojums acquire the characteristics $m_- = 1, A_1 = \sin^2(\alpha_0/2), m_+ = 1, A_2 = -\sin^2(\alpha_0/2)$. Two islands with different signs of $\mathbf{n}\cdot\mathbf{v}$ are thus produced on the surface: $\mathbf{n}\cdot\mathbf{v} < 0$ near the source boojum and $\mathbf{n}\cdot\mathbf{v} > 0$ near the sink boojum. This manifests itself in experiment right away in the appearance of an annular disclination that effects the transition between the islands with opposite directions of **n**. As follows from experiment, the disclination remains on the equator until α_0 decreases to 0; the only changes in the systems are the dispersal of the boojums with $A_1 = -A_2 = \sin^2(\alpha_0/2)\to 0$ and the enhancement of the disclination (cf. Figs. 7a–7c). It must be noted here that the dispersal and enhancement of the defects, according to the theory, must be accompanied by a corresponding increase or decrease of their cores (6) and (11). This is confirmed by experiment, but quantitative measurements are made difficult because these quantities amount to only a few microns.

It appears that the reason why the disclination does not move away from the equator at $\alpha_0 \neq 0$ is its interaction with the boojums (repulsion). Since the boojums have equal values of A, their repulsions are equal and as a result the equilibrium position of the disclination is on the equator. The existence of repulsion between a boojum and a disclination is confirmed also by the process described in §3 below.

The situation changes at $\alpha_0 = 0$ (Fig 7d). Both boojums disperse by that time to form a uniform distribution, $m_+ = m_- = 0$, $A_1 = A_2 = 0$ with, according to (4) $N_1 = N_2 = 0$. The conservation laws (13) hold in this case. In the absence of boojums, the contraction of the disclination into a point on the surface becomes energywise preferred, since the only consequence of this process is a decrease of the disclination length. The disclination contracts, preserving the form of a flat ring. In accord with (12), its topological characteristic A_d varies in this case like $Ad = \sin\beta$ from 0 to 1 (A_d is taken with a plus sign, since the island with $\mathbf{n}\cdot\mathbf{v} < 0$ is cancelled). As a result of the contraction we get a point hedgehog on the surface (Fig. 7e) with $N = 1$, which goes off subsequently to the interior of the drop (Fig. 7f). The second of the conservation laws (9) is then satisfied.

From the topological point of view, the replacement of



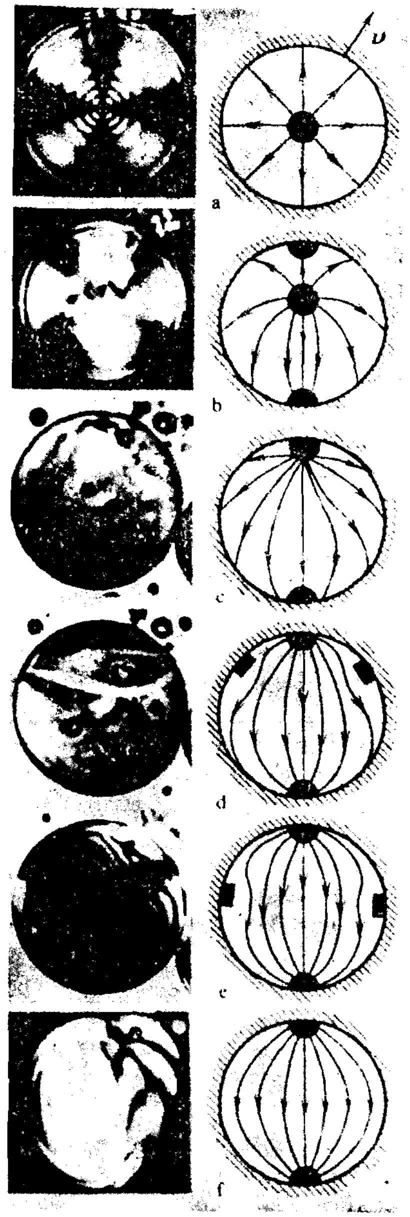


FIG. 7. Topological dynamics of defects in a nematic drop on going from tangential to normal boundary conditions. First column—microphotographs of textures of one of the same drop of 32 μm radius, second—schemes of corresponding distributions of the director fields. The cores of the boojums, hedgehogs, and disclinations are marked respectively by semicircles, circles, and rectangles. The microphotographs in Figs. a, b, and f were taken with crossed nicols, the others without nicols.
a) $\alpha_0 = \pi/2$; two boojums on the poles with respective charges $m_1 = 1$, $A_1 = 1/2$, $N = 1$ and $m_2 = 1$, $A_2 = -1/2$, $N_2 = 0$; there are no hedgehogs or disclinations.
b) $\alpha_0 \lesssim \pi/2$; a disclination with charge $A_d = 0$ appears on the equator "out of nowhere"; the boojum charges are respectively $m_1 = 1$, $A_1 = \sin^2(\alpha_0/2)$, $N_1 = 1$ and $m_2 = 1$, $A_2 = \sin^2(\alpha_0/2)$, $N_2 = 0$.
c) $\alpha_0 = 0$; the boojums were smoothly annihilated after which the disclination shifts towards the north pole; its charge increases like $A_d = \sin\beta \to 1$.
e) $\alpha_0 = 0$; disclination contracted into a hedgehog on the north pole, hedgehog charge $N = A_d = 1$.
f) $\alpha_0 = 0$; the only defect in the system is a hedgehog ($N = 1$) which went over from the surface into the interior.

FIG. 8. Topological dynamics of defects in a nematic drop of radius 37 μm on going from normal to tangential boundary conditions. The notation is the same as in Fig. 7. The microphotographs a, b, and f were obtained with crossed nicols, c and d with parallel and e with obliquely oriented nicols.
a) $\alpha_0 = 0$; hedgehog with charge $N = 1$ at the center of the drop; there are no boojums or hedgehogs.
b) $\alpha_0 = \pi/6$; boojums appear at the poles with respective charges $m_1 = 1$, $A_1 = -\sin^2(\alpha_0/2)$, $N_1 = 0$ and $m_2 = 1$, $A_2 = -\sin^2(\alpha_0/2)$, $N_2 = 0$; hedgehog ($N = 1$) moves towards the northern boojum.
c) $\alpha_0 = \pi/4$; hedgehog and northern boojum coalesce to form a new boojum with $m_1 = 1$, $A_1 = \cos^2(\alpha_0/2)$, $N_1 = 1$; the southern boojum has as before $m_2 = 1$, $A_2 = -\sin^2(\alpha_0/2)$, $N_2 = 0$.
d) $\alpha_0 = \pi/3$; the boojum formed with $A_1 = \cos^2(\alpha_0/2)$ breaks up into a boojum with $m_1 = 1$, $A_1 = \sin^2(\alpha_0/2)$, $N_1 = 1$ and a disclination ring with charge $A_d = \sin\beta\cos\alpha_0$, which moves from the north pole to the equator; the boojum on the south pole has the same characteristics as before.
e) $\alpha_0 \to \pi/2$; the boojums are enhanced, $m_1 = 1$, $A_1 = \sin^2(\alpha_0/2) \to 1/2$, $N_1 = 1$ and $m_2 = 1$, $A_2 = -\sin^2(\alpha_0/2) \to -1/2$, $N_2 = 0$; the disclination vanishes smoothly: $A_d = \sin\beta\cos\alpha_0 \to 0$.
f) $\alpha_0 = \pi/2$; the disclination vanished, a bipolar structure remains with two boojums carrying charges $m_1 = 1$, $A_1 = 1/2$, $N_1 = 1$ and $m_2 = 1$, $A_2 = -1/2$, $N_2 = 0$.

two boojums by one hedgehog is effect by a surface annular disclination: the disclination, arising "from nowhere," is enhanced simultaneously with the dispersal of the boojums, after which it is transformed into a hedgehog. It follows from Fig. 7 that the experimentally observed process agrees fully with the scenario described in §3 of the theoretical part.

On the graphic scheme in Fig. 7f the structure near the hedgehog is represented for simplicity as purely radial. A similar distribution of the field of n is realized in principle in the smectic-A phase 8 and possibly in a narrow region of the nematic phase directly past the transition from smectic-A. In the general case, however, the radial character of the structures in the NLC drops at $\alpha_0 = 0$ is preserved only at distances on the order of several microns from the surface, i.e., over scales at which the action of the surface orientation of n still manifests itself. As for the center of the drop, two basic types of defect distortions can be distinguished here: structures with strong helical twist and structures with linear ring defects (in particular, two disclination rings linked with each other are observed). We note that for an NLC drop taken by itself it would be possible to observe, even at a fixed temperature, transitions of the indicated structures into one another, as well as into more complicated ones, thus attesting to negligible differences between the elastic energies of such distortions. On the whole, however, any of the possible structures has, as follows from the Gauss theorem [see (9)], a charge $N = 1$ and is in fact equivalent to the usual hedgehog.

§4. Transition from radial to bipolar structure

In the defect dynamics described above no account whatever is taken of the possibility of topological interaction of a hedgehog with a boojum as well as of a boojum with a disclination, which might cause the structure and topological charge of the boojums to change jumpwise. Such an interaction is realized on going from normal to tangential boundary conditions (Fig. 8). The gist of the process is the following.

When the vector $\mathbf{n}$ deviates from the direction of the normal, two boojums with $m_1 = m_2 = 1$, $A_1 = A_2 = -\sin^2(\alpha_0/2)$ and, as follows from (4), with $N_1 = N_2 = 0$, appear on the poles (Fig. 8b). Both boojums act as sinks for the field of $\mathbf{n}$. The source is the hedgehog. The latter moves towards one of the boojums (the upper one in Fig. 8b) and merges with it (Fig. 8c). As a result this combination yields a new source boojum with $A_1 = \cos^2(\alpha_0/2)$ and $N_1 = 1$. The latter, however, is unstable and with further increase of α_0 it breaks up into a dislocation ring with $A_d = \sin\beta\cos\alpha_0$ and a source boojum with $A_1 = \sin^2(\alpha_0/2)$ and $N_1 = 1$ (Fig. 8d). The disclination repelled by the boojum moves towards the equator and disperses gradually—simultaneously with the enhancement of the boojums (Fig. 8e). In the final state there remain in the drop two boojums with $A_1 = -A_2 = 1/2$ (Fig. 8f).

Thus, transformations of boojums of various types into one another, as a result of topological interaction of boojums with hedgehogs and with dislocation rings, manifest themselves in the given transition. It can be easily seen that all these processes proceed with conservation of the total topological charges and are subject to the conservation laws derived in theoretical section of the paper.

V. CONCLUSION

We have experimentally observed and theoretically described the topological dynamics of defects under smoothly varying external conditions. When these conditions are varied in a nematic, defects of different topological types (hedgehogs, boojums, dislocations) and of different homotopic classes are smoothly transformed into one another. The defects are characterized by continuous charges A expressed in terms of integer topological charges and a continuous external-conditions parameter (boundary conditions). When the boundary conditions are changed, the charges A become continuously redistributed among the defects, with the total charge in the drop conserved. The mutual transformation of the defects takes place when A goes through an integer value; for example, the boojums are transformed into hedgehogs at $A = 1$. As $A \to 0$ continuous annihilation of a defect takes place, accompanied by an increase of the size of its core to infinity. We assume that these features are typical of the topological dynamics of defects in other physical systems.

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